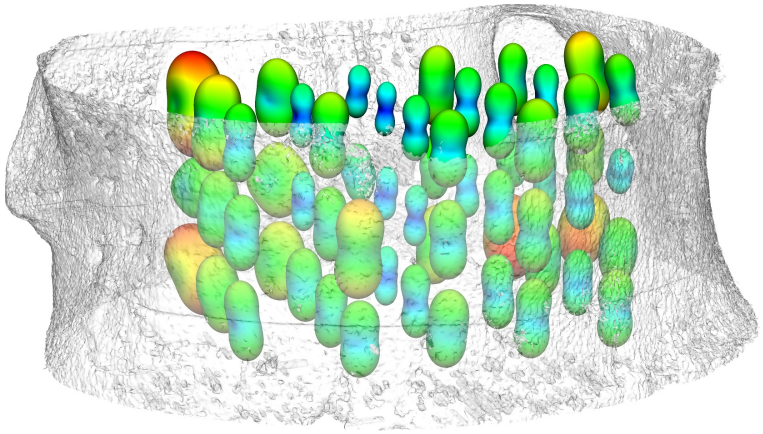
MULTISCALE ELASTICITY MODELING OF TRABECULAR BONE

Lecture Series on Reductionism in Physics and its Relation to Philosophy
Jacobs University, Spring 2013

Lars Ole Schwen
Fraunhofer MEVIS

2013-03-08



Fraunhofer MEVIS

1. Introduction

Overview

- example of multiscale modeling
- biomechanical problem
- mathematical simulation tools

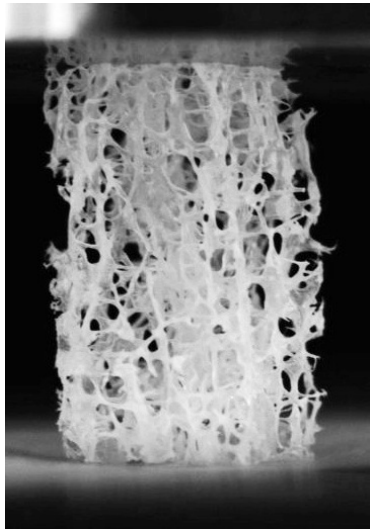
Cooperation Partners:

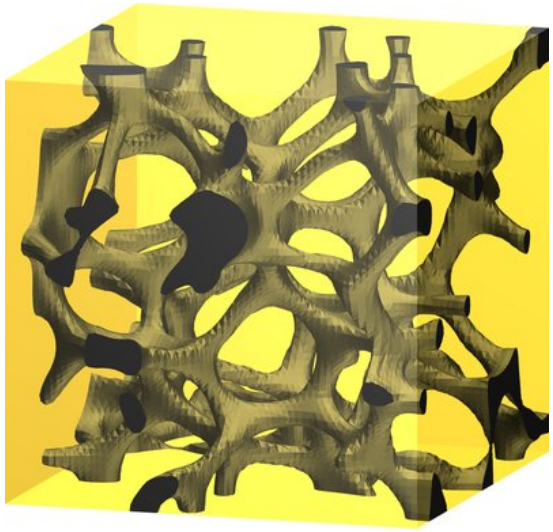
Martin Rumpf (INS, U Bonn)
Torben Pätz (Jacobs U and MEVIS)
Tobias Preusser (Jacobs U and MEVIS)
Stefan Sauter (U Zurich)
Hans-Joachim Wilke (UFB, U Ulm)
Uwe Wolfram (UFB, U Ulm and U Bern)

MULTISCALE ELASTICITY MODELING OF TRABECULAR BONE

2. Trabecular Bone

Specimens

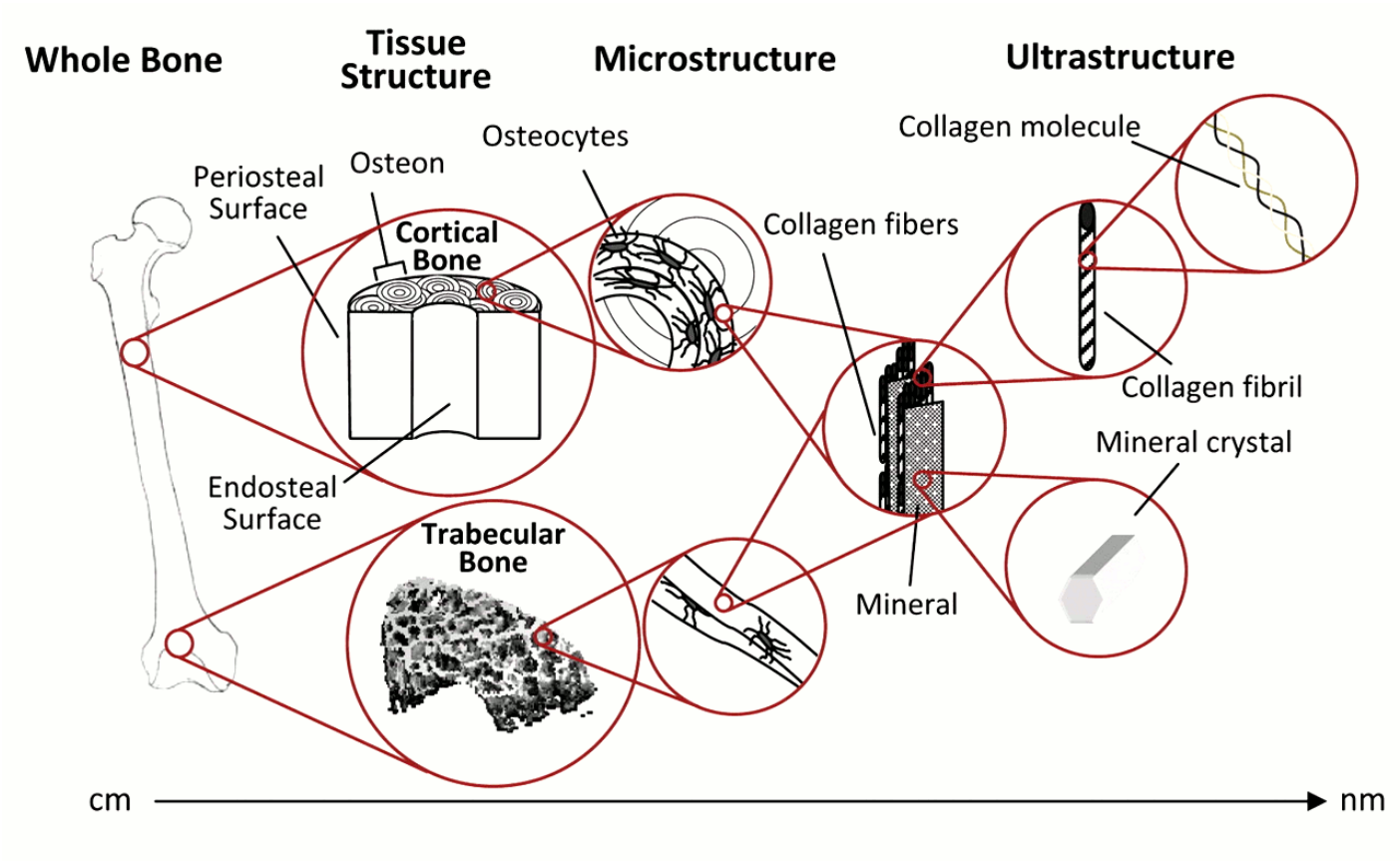




Specimens of trabecular bone obtained from vertebrae

2. Trabecular Bone

Scales



A. Boskey and R. Coleman, Journal of Dental Research 89 (2010), pp. 1333–1348
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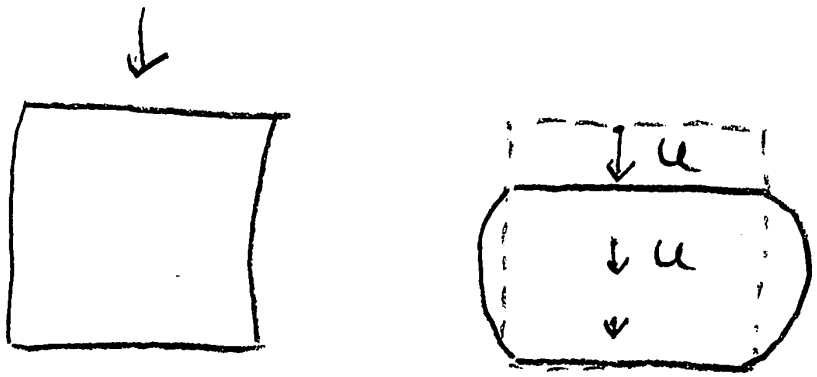
3. Modeling Linearized Elasticity

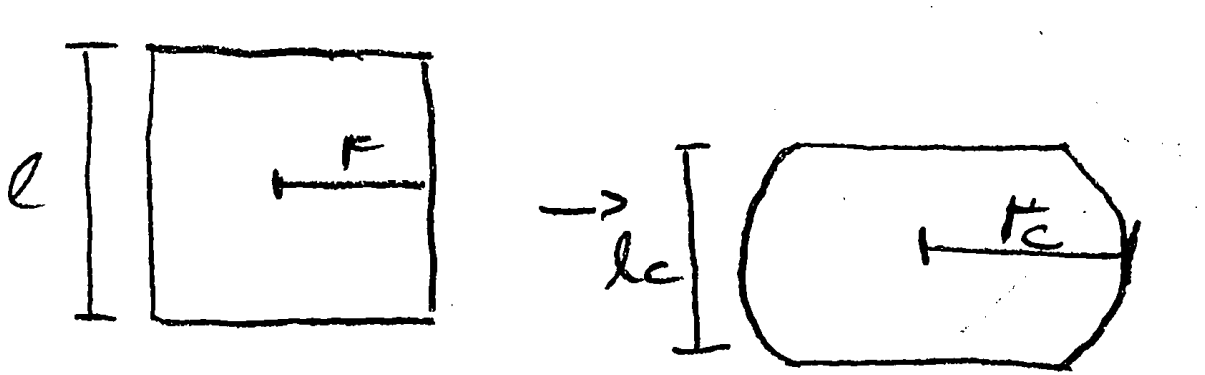
Basics of Linearized Elasticity



- **Displacement** deformation of elastic body described by displacement $u : \Omega \to \mathbb{R}^3$
  Where is a point shifted to?
- **Rigid body motions** translation or rotation does not deform object
- **Strain** "symmetrized gradient" $\epsilon = \frac{1}{2}\left(\nabla u + \nabla u^T\right) \in \mathbb{R}^{3\times 3}$
  1D: How much is a spring compressed?
- **Stress** related to strain via elasticity tensor, $\sigma = C : \epsilon$, $C \in \mathbb{R}^{3\times 3\times 3\times 3}$
  1D: force needed for compressing spring

3. Modeling Linearized Elasticity

Basics of Linearized Elasticity



Simplest case of elasticity tensor C

- **Young's modulus** E describes stiffness
- **Poisson's ratio** ν describes bulging, i.e. coupling between directions
  no 1D analogon

3. Modeling Linearized Elasticity

Voigt Notation

- strain tensor ϵ is symmetric due to physical reasons, i.e. have 6 instead of 9 entries
- same for stress tensor σ

$\Rightarrow$ can write C as symmetric 6×6 matrix with 21 constants

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} & C_{04} & C_{05} \\ & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ & & C_{22} & C_{23} & C_{24} & C_{25} \\ & & & C_{33} & C_{34} & C_{35} \\ & \text{sym} & & & C_{44} & C_{45} \\ & & & & & C_{55} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ 2\epsilon_{yz} \\ 2\epsilon_{xz} \\ 2\epsilon_{xy} \end{bmatrix}$$

3. Modeling Linearized Elasticity

Isotropy to Anisotropy

- **isotropic** all directions are equivalent
  two constants (E and ν)
- **transversely isotropic** one direction is "special", the two perpendicular directions are equivalent
- **orthotropic** three perpendicular directions are pairwise different
- …
- **fully anisotropic** most general case
  21 constants

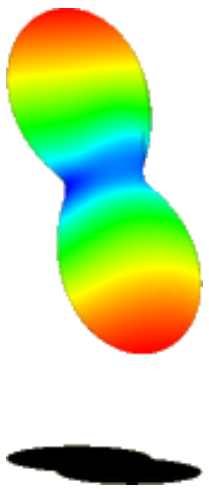
$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} C_{00} & C_{01} & C_{02} & C_{03} & C_{04} & C_{05} \\ & C_{11} & C_{12} & C_{13} & C_{14} & C_{15} \\ & & C_{22} & C_{23} & C_{24} & C_{25} \\ & & & C_{33} & C_{34} & C_{35} \\ & \text{sym} & & & C_{44} & C_{45} \\ & & & & & C_{55} \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{zz} \\ 2\epsilon_{yz} \\ 2\epsilon_{xz} \\ 2\epsilon_{xy} \end{bmatrix}$$

3. Modeling Linearized Elasticity

Visualization of Elasticity Tensors

6×6 matrix easier to understand than fourth order tensor $C \in \mathbb{R}^{3\times3\times3\times3}$, but still …

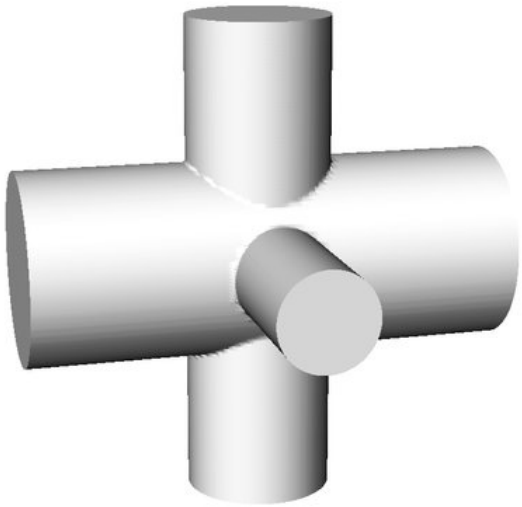
Tensors can be visualized as deformed (encoding uniaxial stiffness) and colored (encoding bulk modulus) spheres (“colored peanuts”).

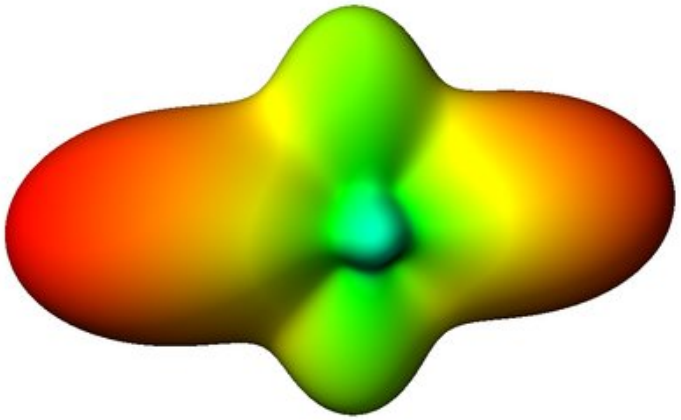


[He et al., Int J Solids and Structures 10(1995),
Cazzani et al., Int J Solids and Structures 40(2003)]

3. Modeling Linearized Elasticity

Visualization of Elasticity Tensors, Examples

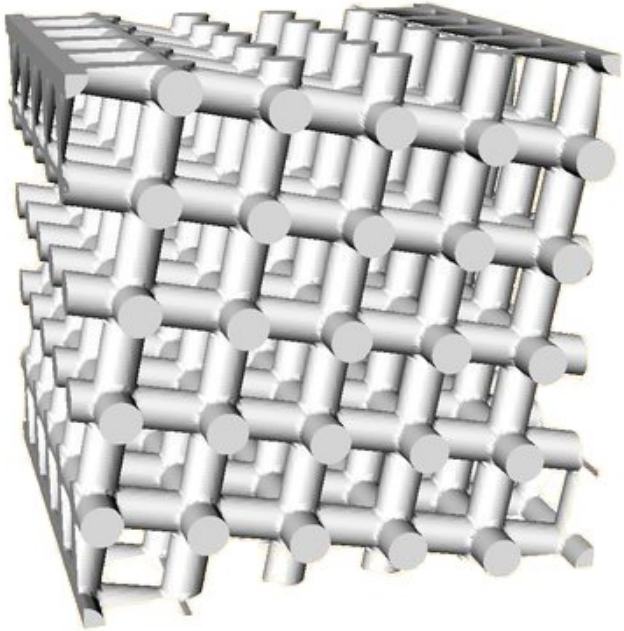


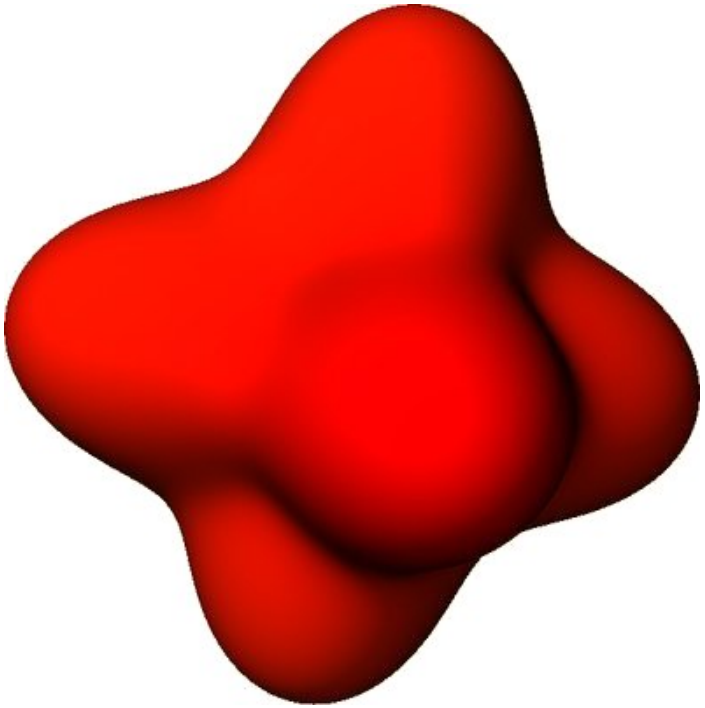


macroscopically orthotropic (⇐ geometry + microscopic isotropy)

3. Modeling Linearized Elasticity

Visualization of Elasticity Tensors, Examples

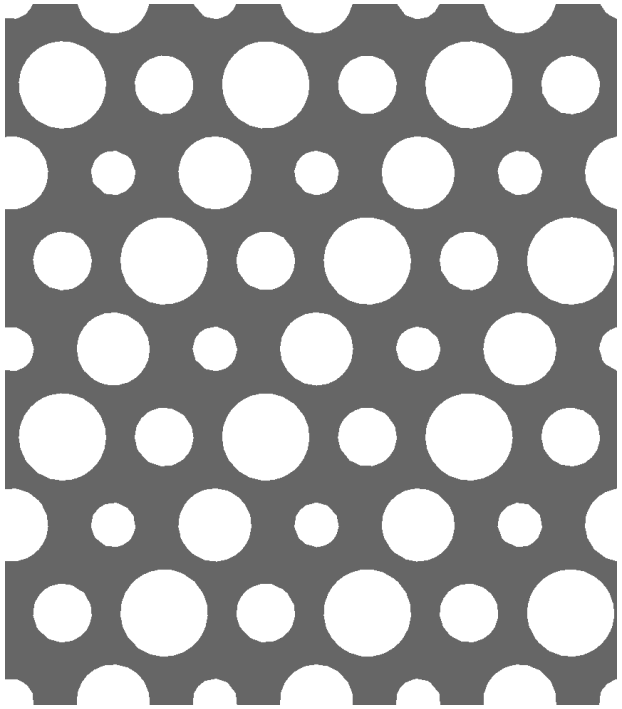


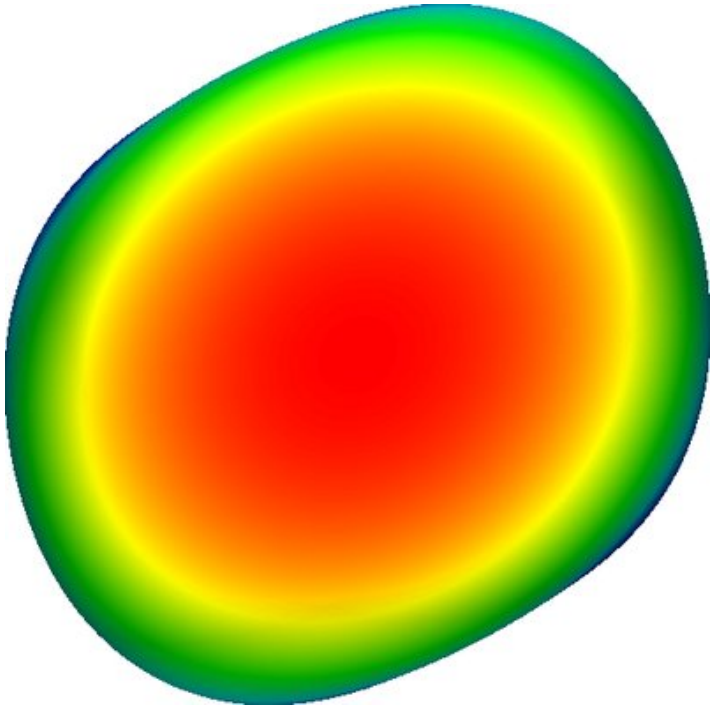


rotated – identify axes!

3. Modeling Linearized Elasticity

Visualization of Elasticity Tensors, Examples



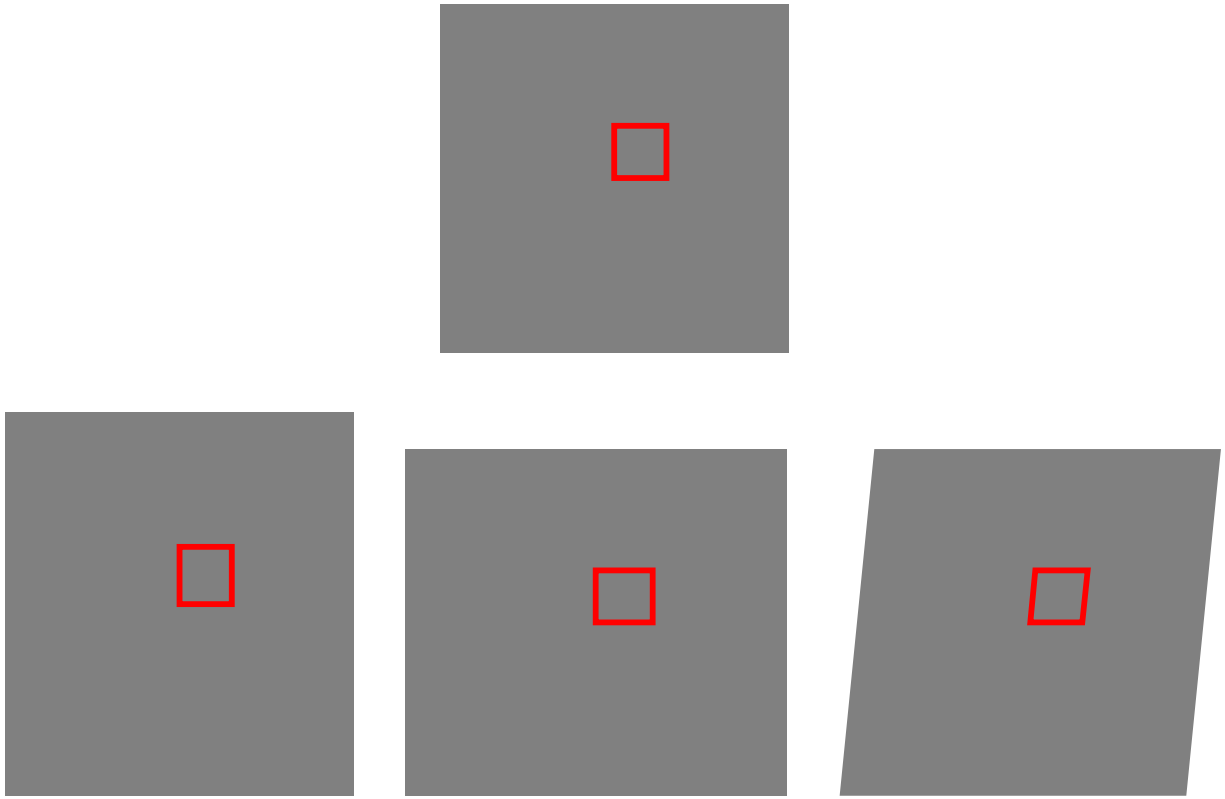


non-orthotropic (but I don’t know how many constants are really necessary)

4. Numerical Homogenization for Multiscale Modeling

Outline

Determine effective elasticity tensor by **imposing** different **unit strains** and **computing stress response**.



4. Numerical Homogenization for Multiscale Modeling

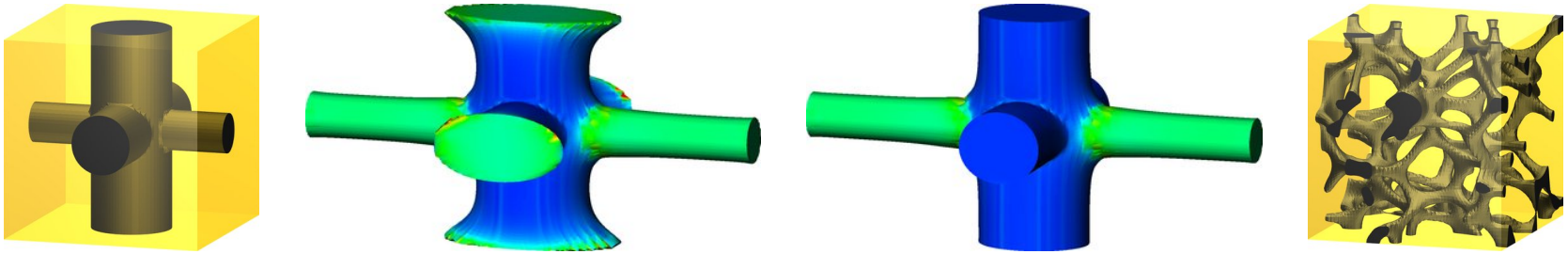
Statistically Periodic Objects

In case of non-periodic microstructures, need statistically representative fundamental cell of at least 5 inter-trabecular distances.

[Harrigan et al, JBiomech 21:269]

Problem

- artificial stiffening near the boundary by enforcing displacement boundary conditions (middle)



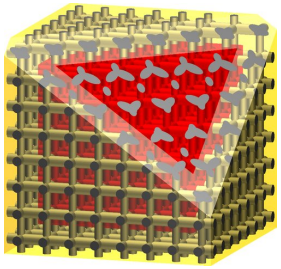
- cannot impose periodic boundary conditions on non-periodic objects

4. Numerical Homogenization for Multiscale Modeling

Statistically Periodic Objects

Solution

- ignore boundary layer β for stress evaluation
- evaluate stress only in interior



- typically $\beta = 12.5\ \%$ cf. [Ün, Bevill, Keaveny J Biomech 39:1955]
- interior still needs to be representative
- trade-off between accuracy and workload

5. From Experiment To Model

Measuring Elasticity Properties

Elasticity Tensor on the trabecular scale measured using **nanoindentation**

- small pyramid-shaped object pressed into trabecula
- allows calculating elasticity properties of trabeculae

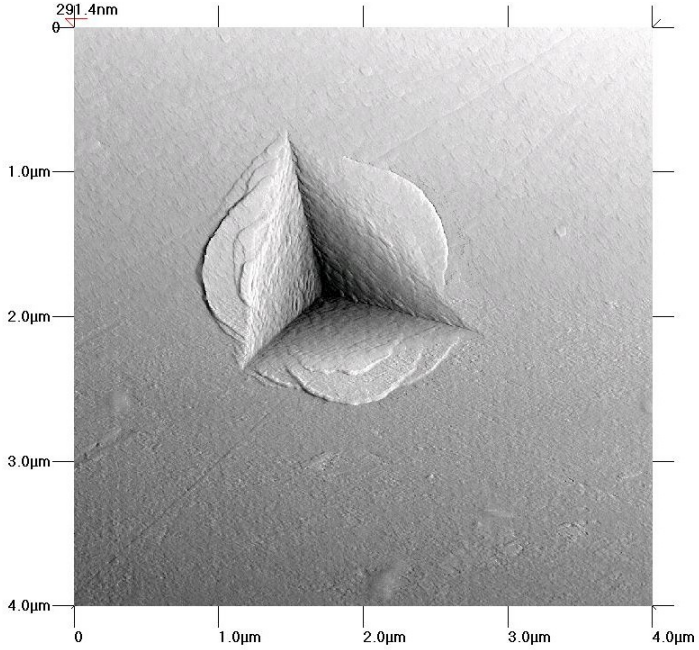


image by Jonathan Puthoff from http://en.wikipedia.org/wiki/File:Nanoindent.JPG

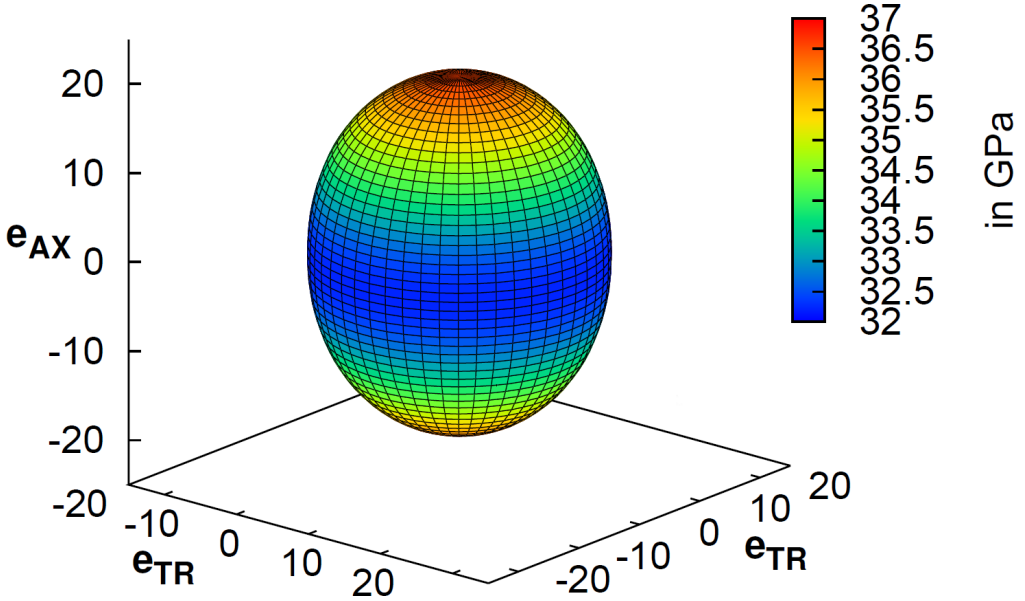
5. From Experiment To Model

Measuring Elasticity Properties

Result

U. Wolfram, H. J. Wilke, P. Zysset, J. Biomech. 43 (2010), 1731–37

- obtain transversely isotropic tensor



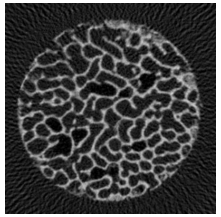
- we didn't determine trabecular orientation but only used isotropic tensor

5. From Experiment To Model

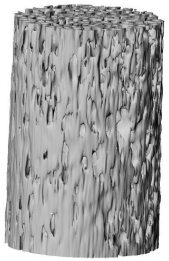
3D Imaging

↓ extract

↓ µCT scan



↓ segment



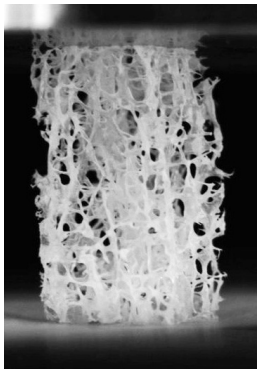
- drill cylindrical specimen
- scan in µCT
- image processing (segmentation) to obtain geometry information

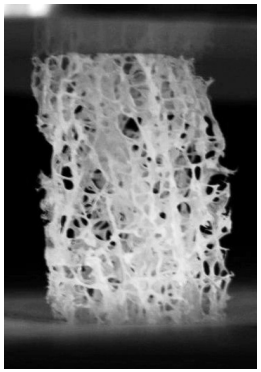
5. From Experiment To Model

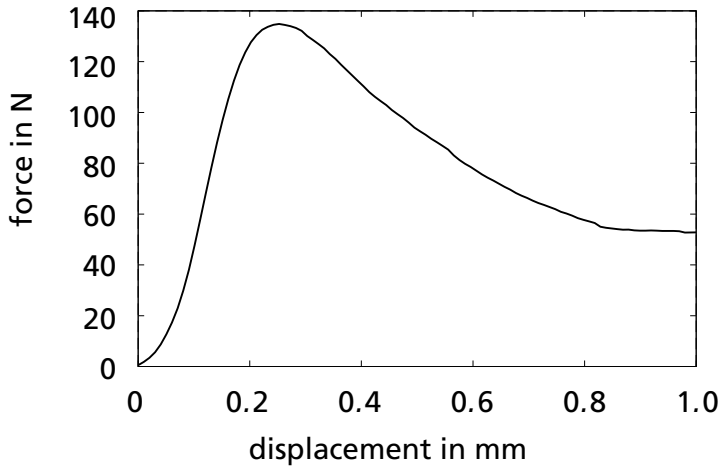
Elasticity Simulation and Validation

- simulation using Composite Finite Elements
- validated experimentally

⇓







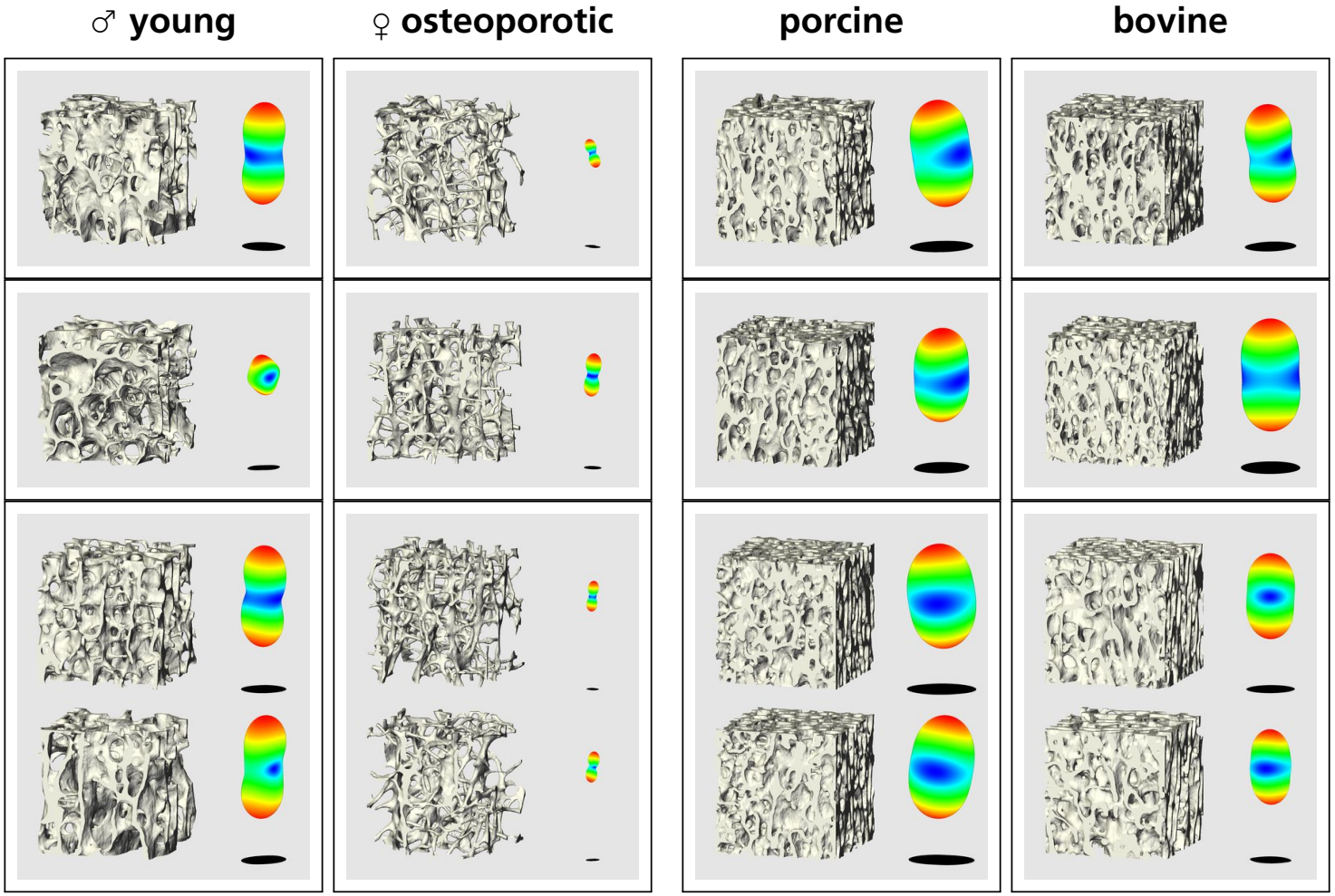
compare simulated uniaxial stiffness to experimentally measured stiffness in linear range

L. O. Schwen, U. Wolfram, Comp. Meth. Biomech. Biomed. Eng, to appear

So let's apply it for obtaining effective elasticity tensors …

5. From Experiment To Model

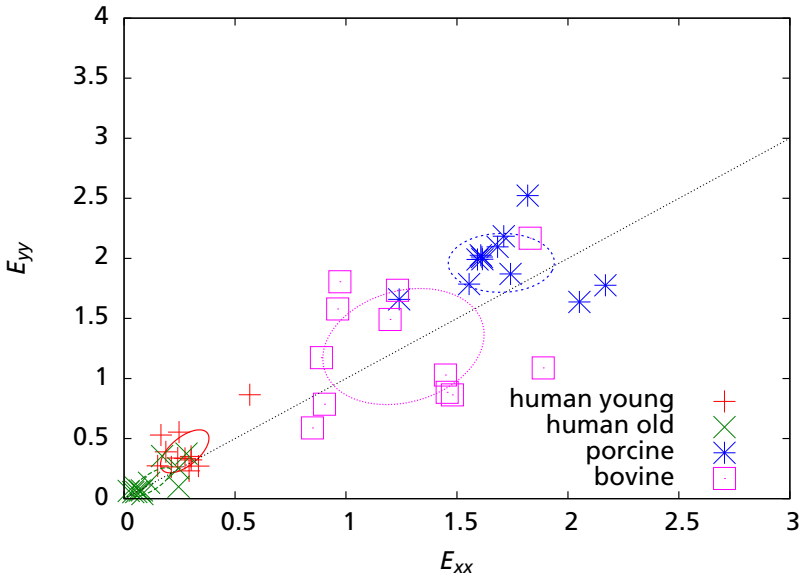
Results I: Differences between Species

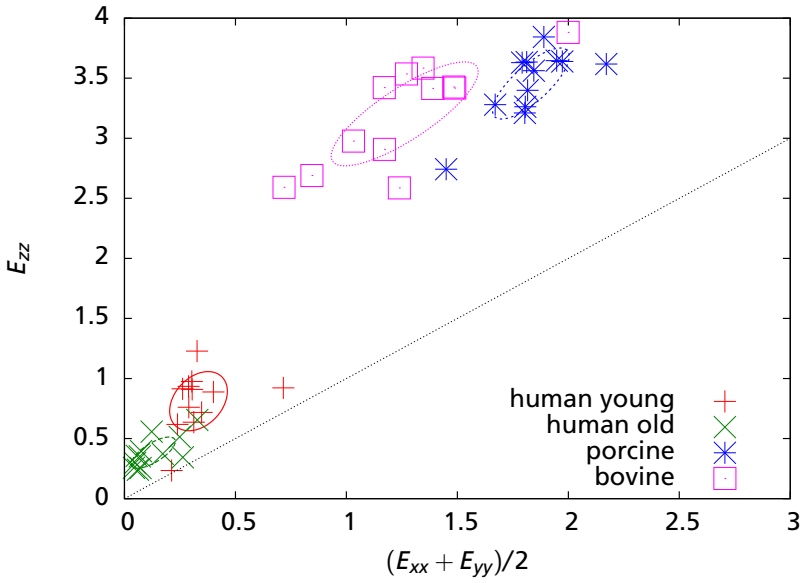


human tensors scaled by 4

5. From Experiment To Model

Results I: Differences between Species





(ellipses: 1 standard deviation obtained by PCA)

Interpretation:

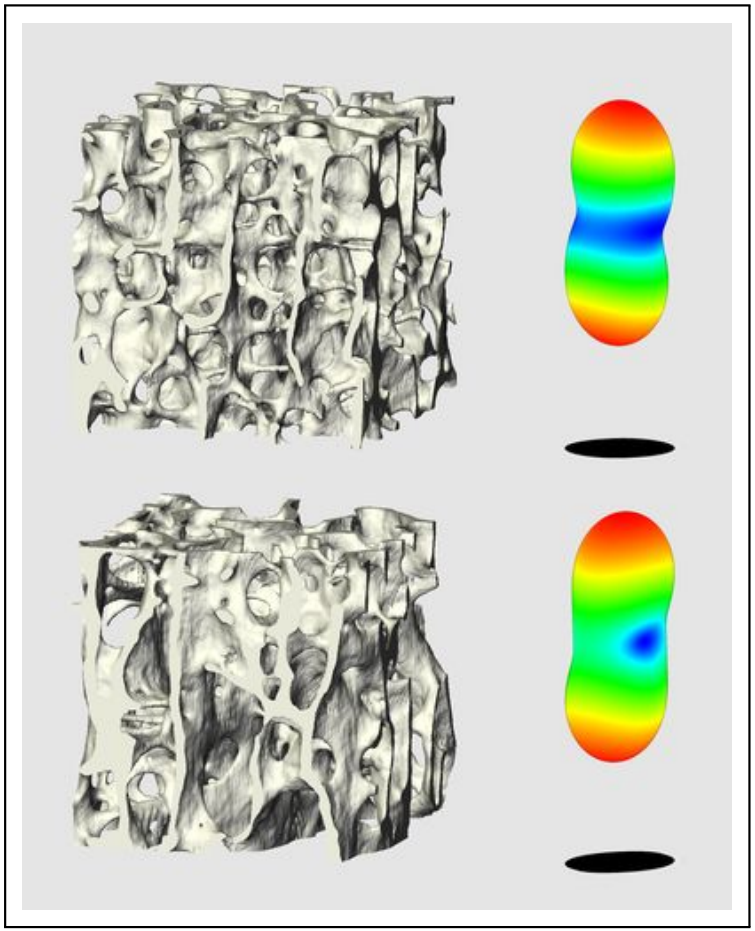
- transverse isotropy?
- anisotropy: craniocaudal/average transverse
- distinct clusters

5. From Experiment To Model

Results II: Intra-Specimen Variations

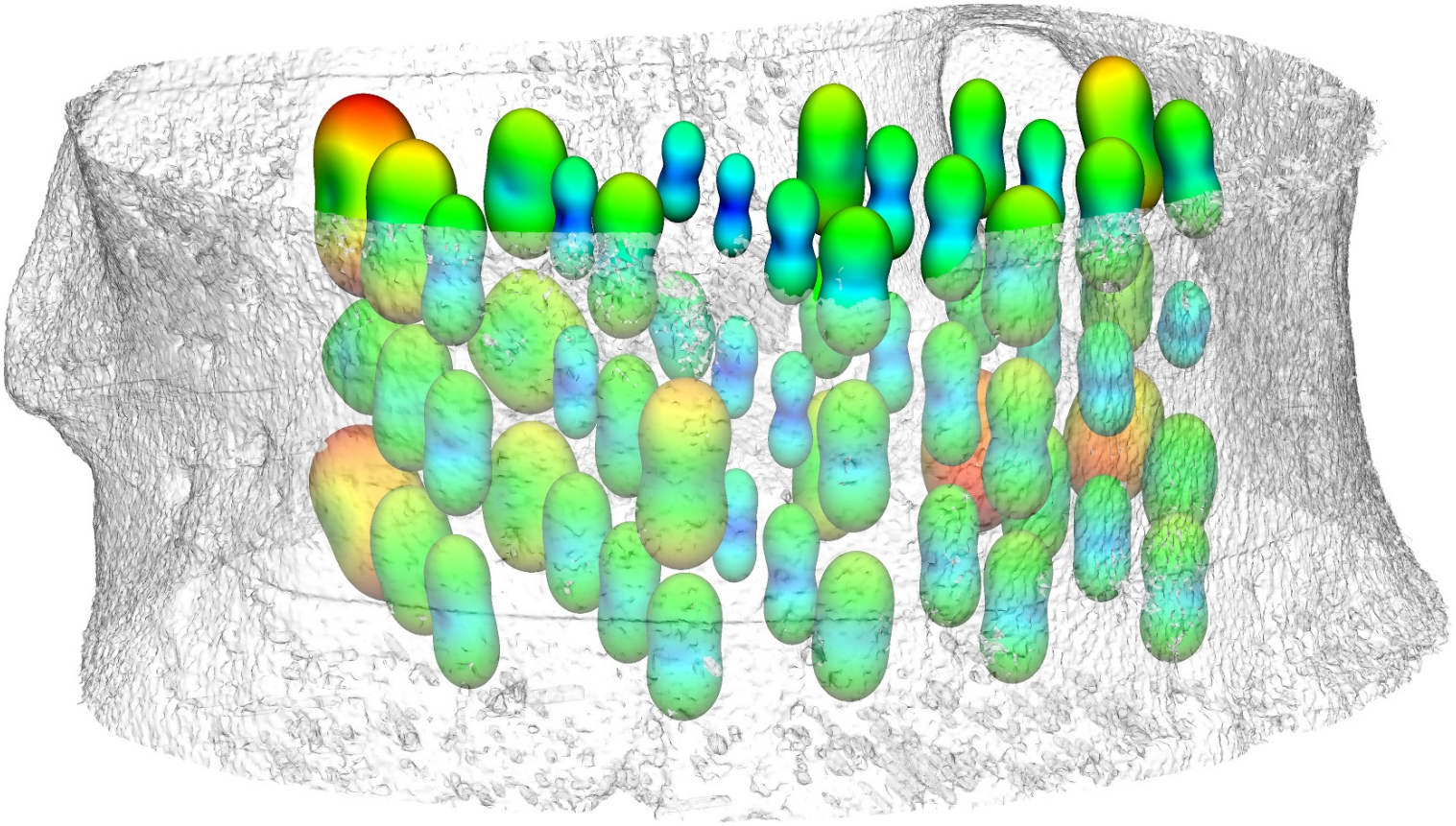
♂ young example from above

- effective elasticity tensor varies in space
- can also consider a whole vertebra



5. From Experiment To Model

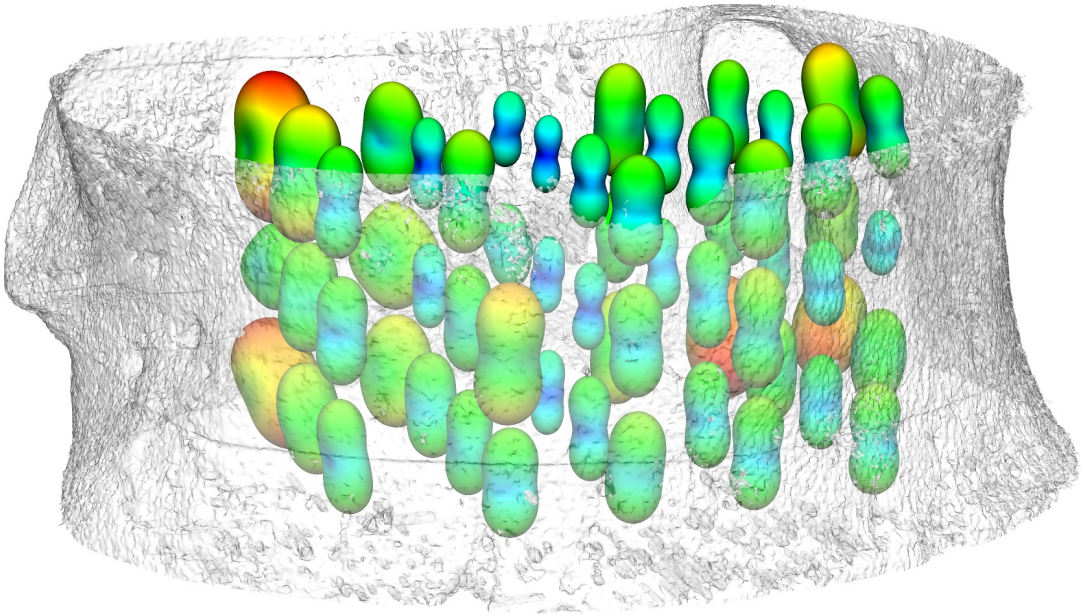
Results II: Intra-Specimen Variations



human ♀ L4 vertebra

5. From Experiment To Model

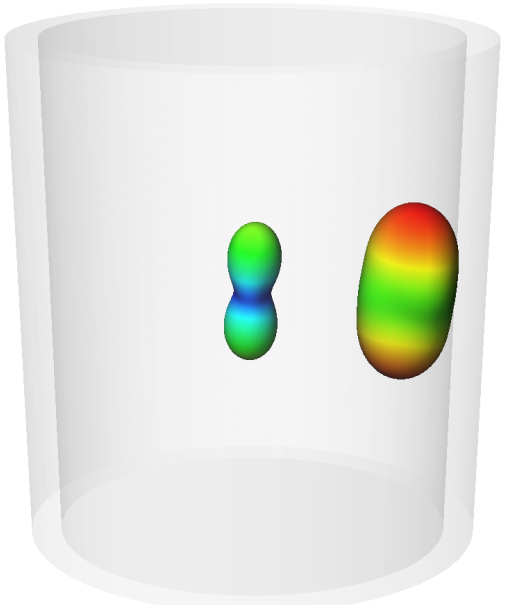
Two-Scale Simulation



- continuously varying elasticity tensor in trabecular interior (spongiosa)
- higher stiffness in exterior wall (compacta)
- CFE elasticity simulation

5. From Experiment To Model

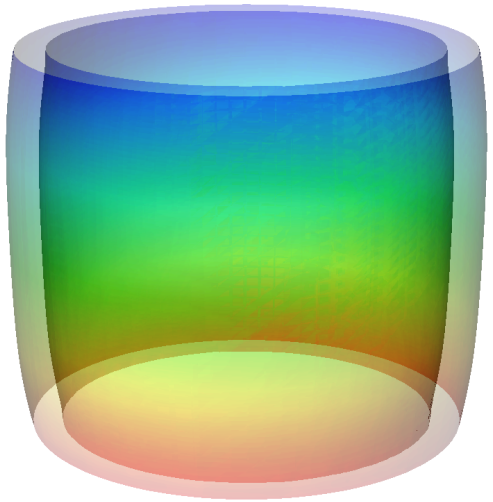
Two-Scale Simulation: Model Geometry



- continuously varying elasticity tensor in trabecular interior (spongiosa)
- higher stiffness in exterior wall (compacta), “peanut” not shown
- CFE elasticity simulation

5. From Experiment To Model

Two-Scale Simulation



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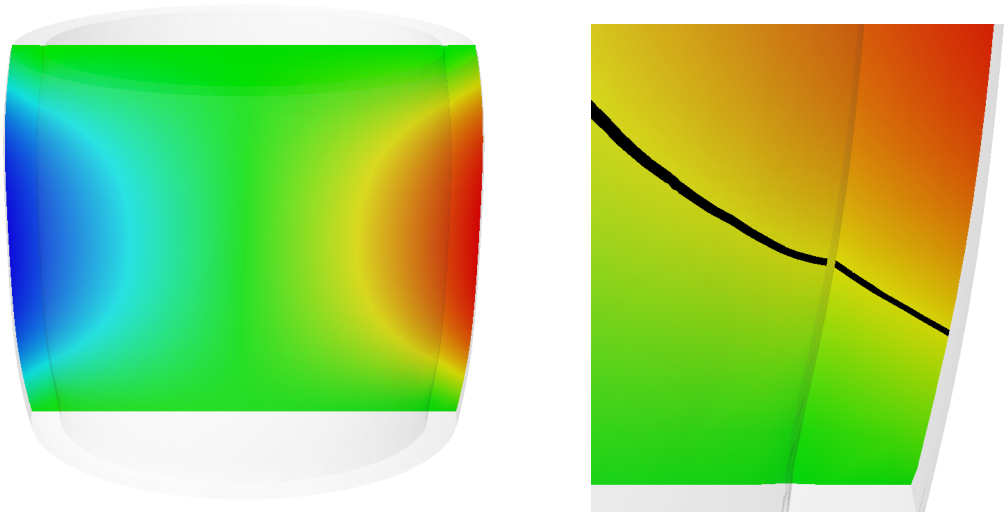

- continuously varying elasticity tensor in trabecular interior (spongiosa)
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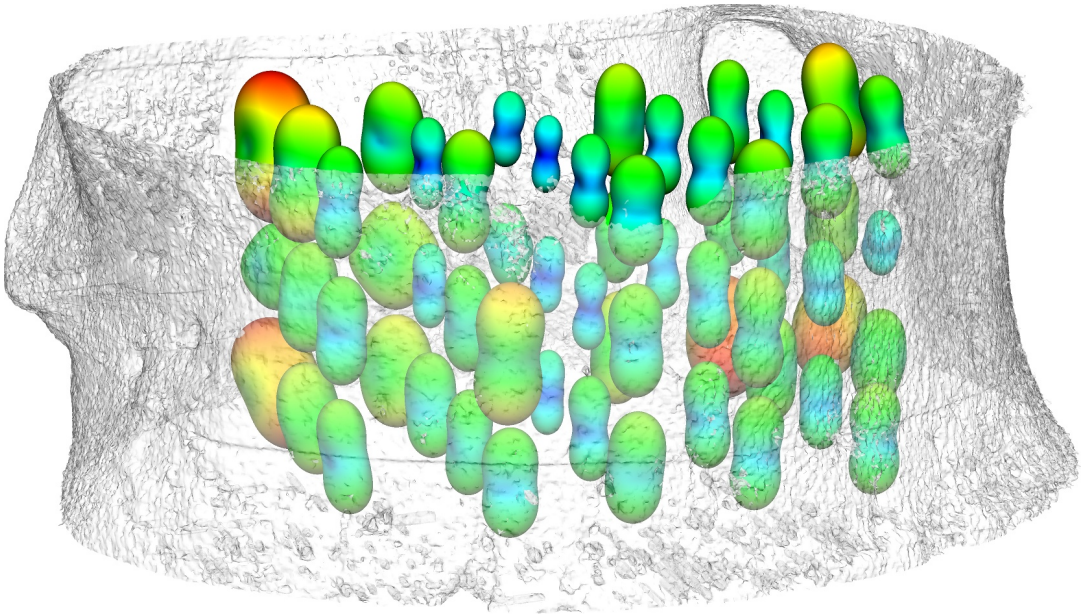
5. From Experiment To Model

Outlook



- proper two-scale model
- taking into account trabecular orientation
  (more realistic properties on the fine scale)

6. Summary

Elasticity Modeling of Trabecular Bone

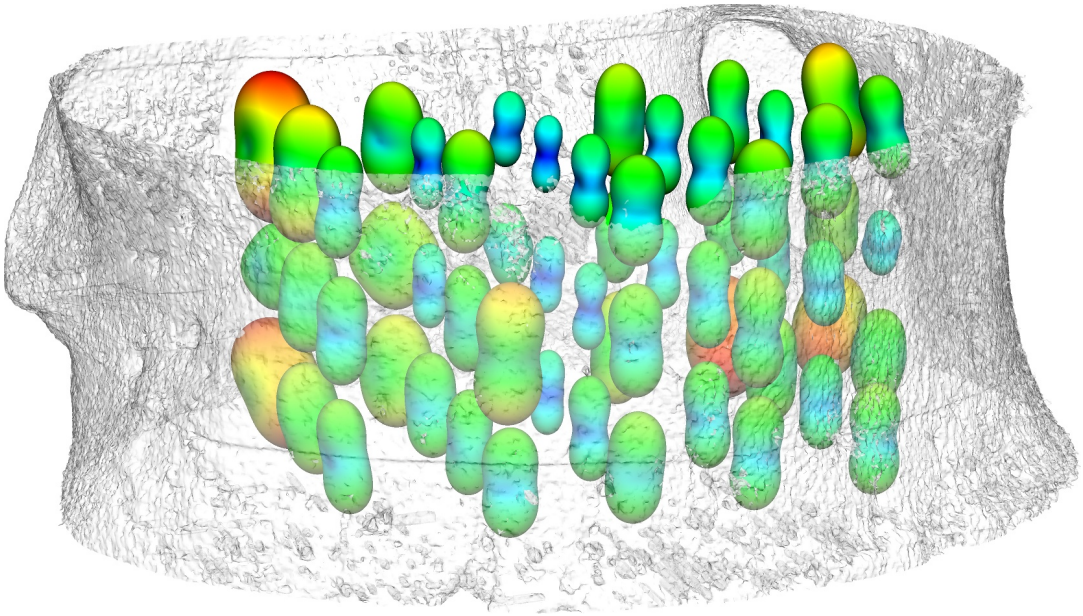
- modeling and simulations require
  - experimental input data and
  - experimental validation
- trabecular bone has multiple scales
- homogenization yields effective material properties for coarser scales

6. Summary

Buzzword Check

- ✔ **interdisciplinary** biomechanical experiments & mathematical simulation tools
- ✔ **nano** we did nanoindentation
- ✘ **real-time** unfortunately, we're far from that in terms of computational efficiency
- ✘ **sustainable**
- ■ **web 2.0** open source, but not more

6. Summary



Contact: Ole Schwen ‹ole.schwen@mevis.fraunhofer.de›