COMPOSITE FINITE ELEMENT SIMULATION OF RADIO FREQUENCY ABLATION AND BONE ELASTICITY

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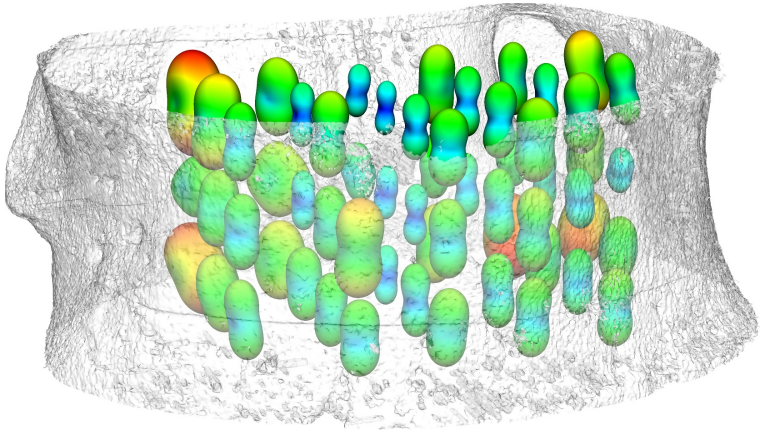
European Congress on Computional Methods
in Applied Sciences and Engineering 2012
Vienna, 2012-09-13



Fraunhofer MEVIS

Outline

1. Introduction

2. Composite Finite Elements

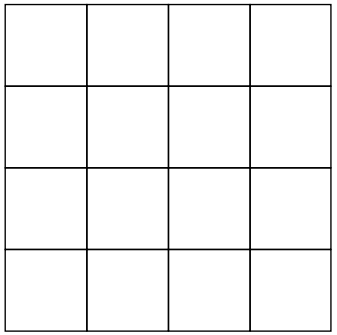
3. Applications

1. Introduction

Why Composite FE?

Classical Finite Elements

- structured cubic grids (cf. voxel images)
  - ✔ efficient
  - ✔ natural coarse scales
  - ✘ poor geometric flexibility



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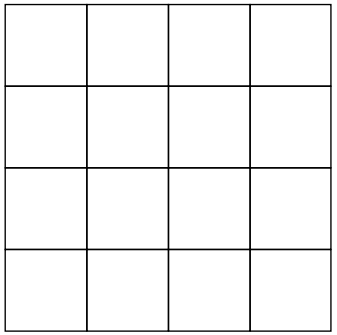

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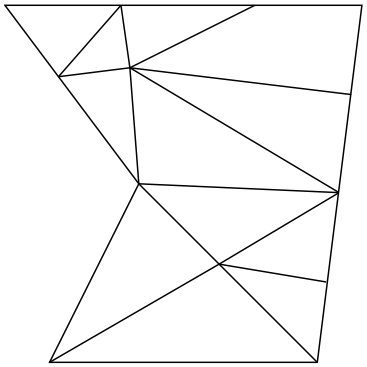
  - ✔ high geometric flexibility
  - ✘ meshing necessary
  - ✘ no natural coarse scales



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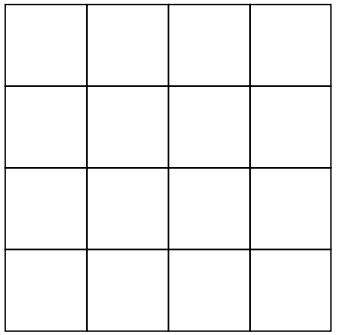

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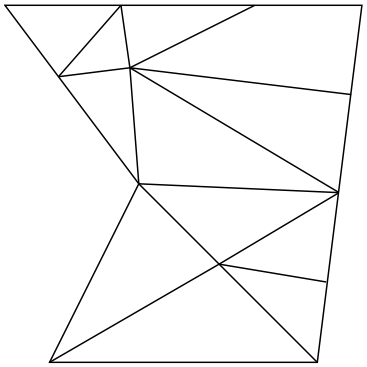

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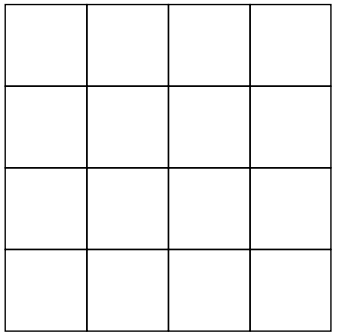

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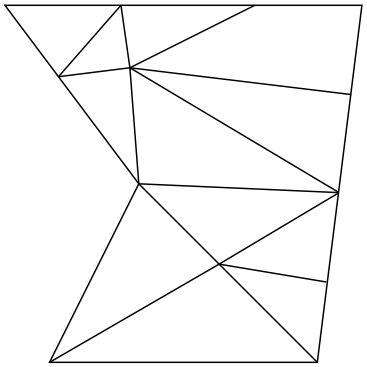

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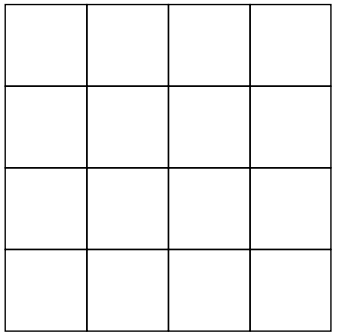

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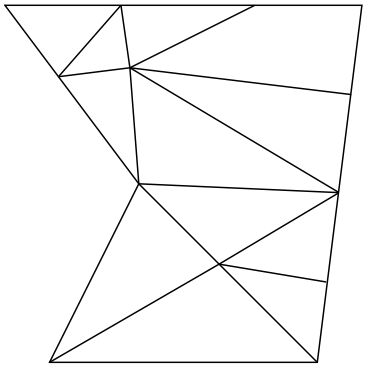
  - key property: **complicated grid with simple basis functions**



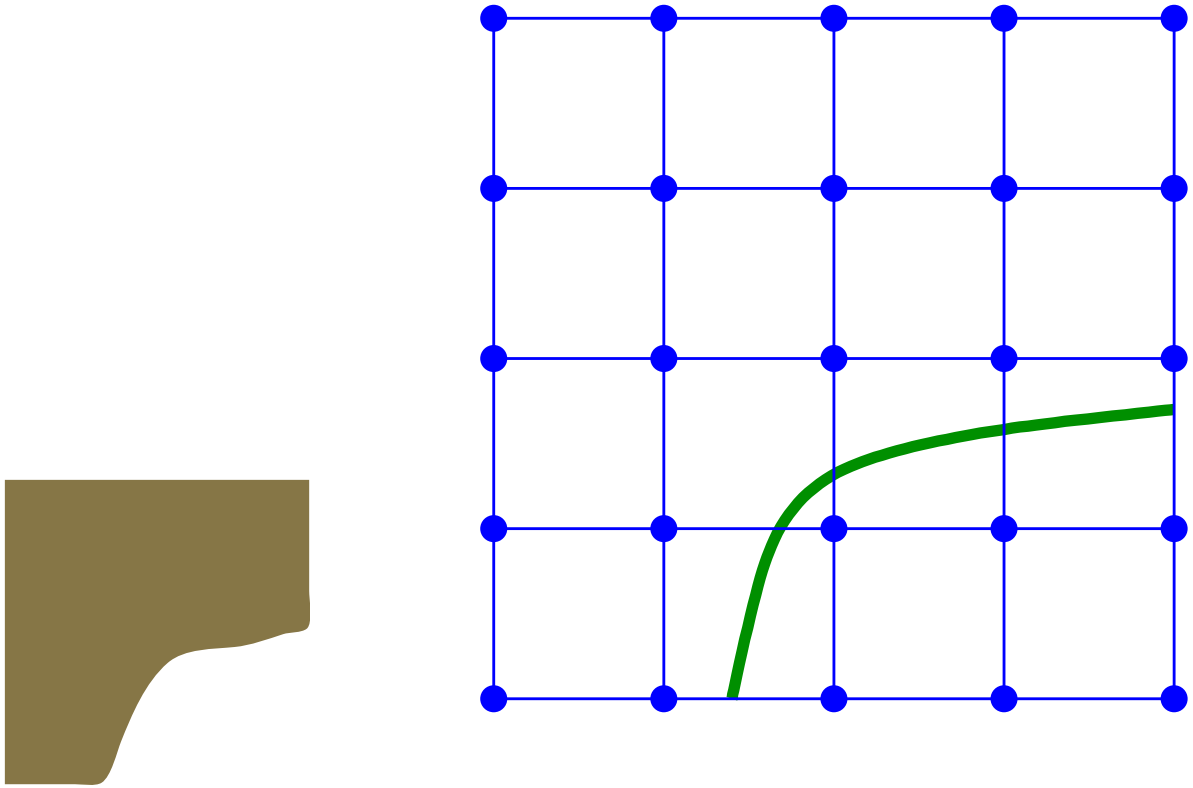
Composite Finite Elements

- combine advantages of both
- use **simple grid with complicated basis functions**

2. Composite Finite Elements

CFE for Geometrically Complicated Domains (2D)

3D: [Liehr et al., Comput Vis Sci 2009]



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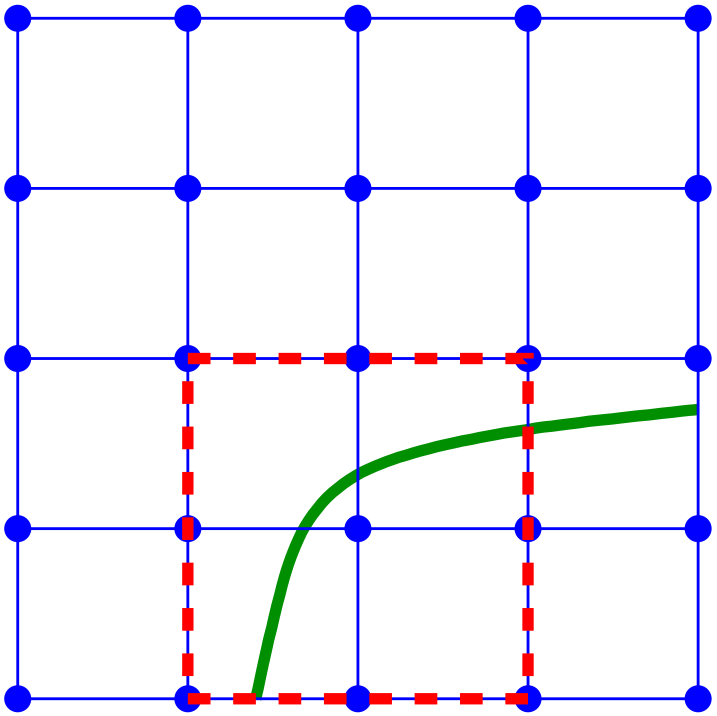

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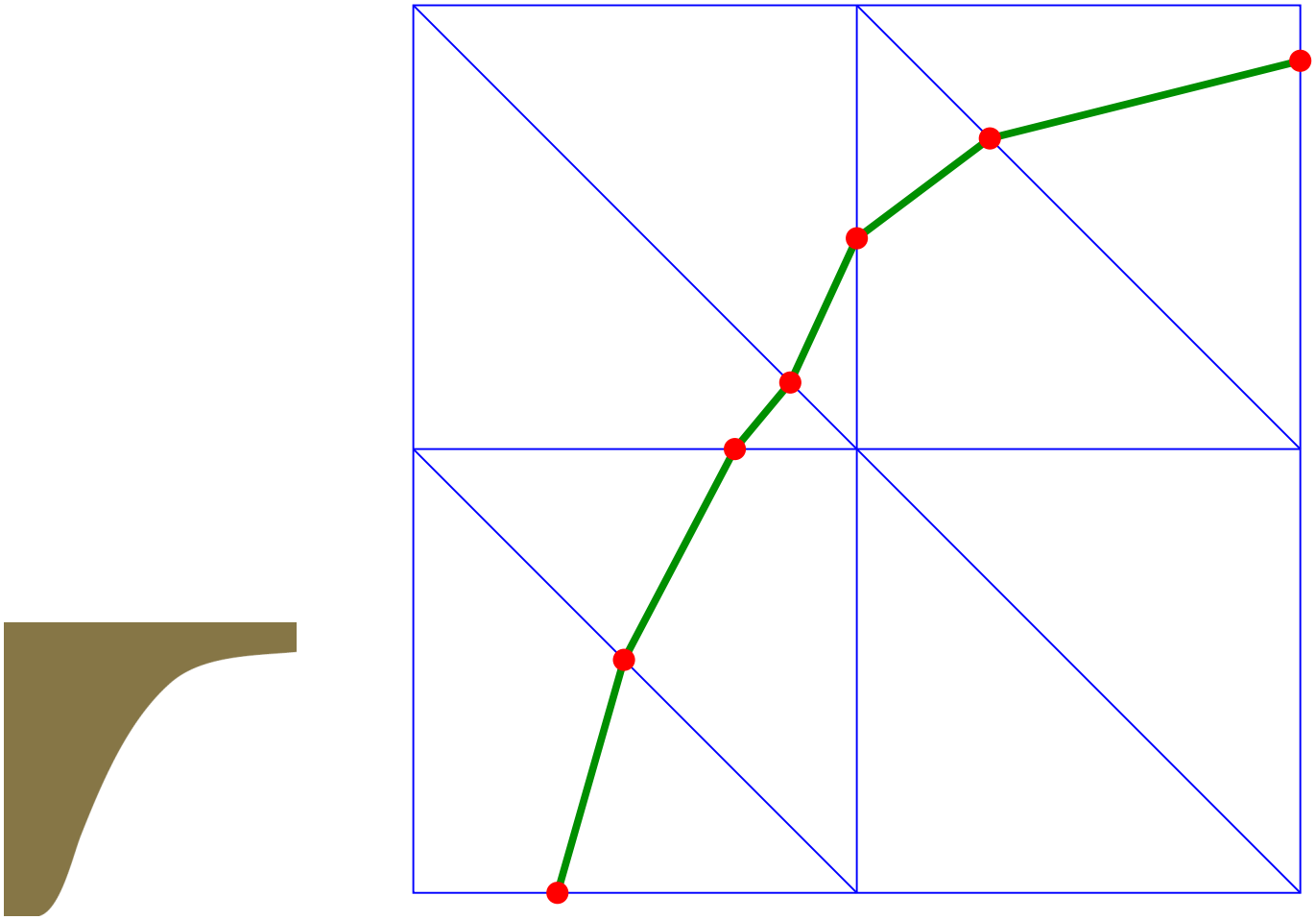

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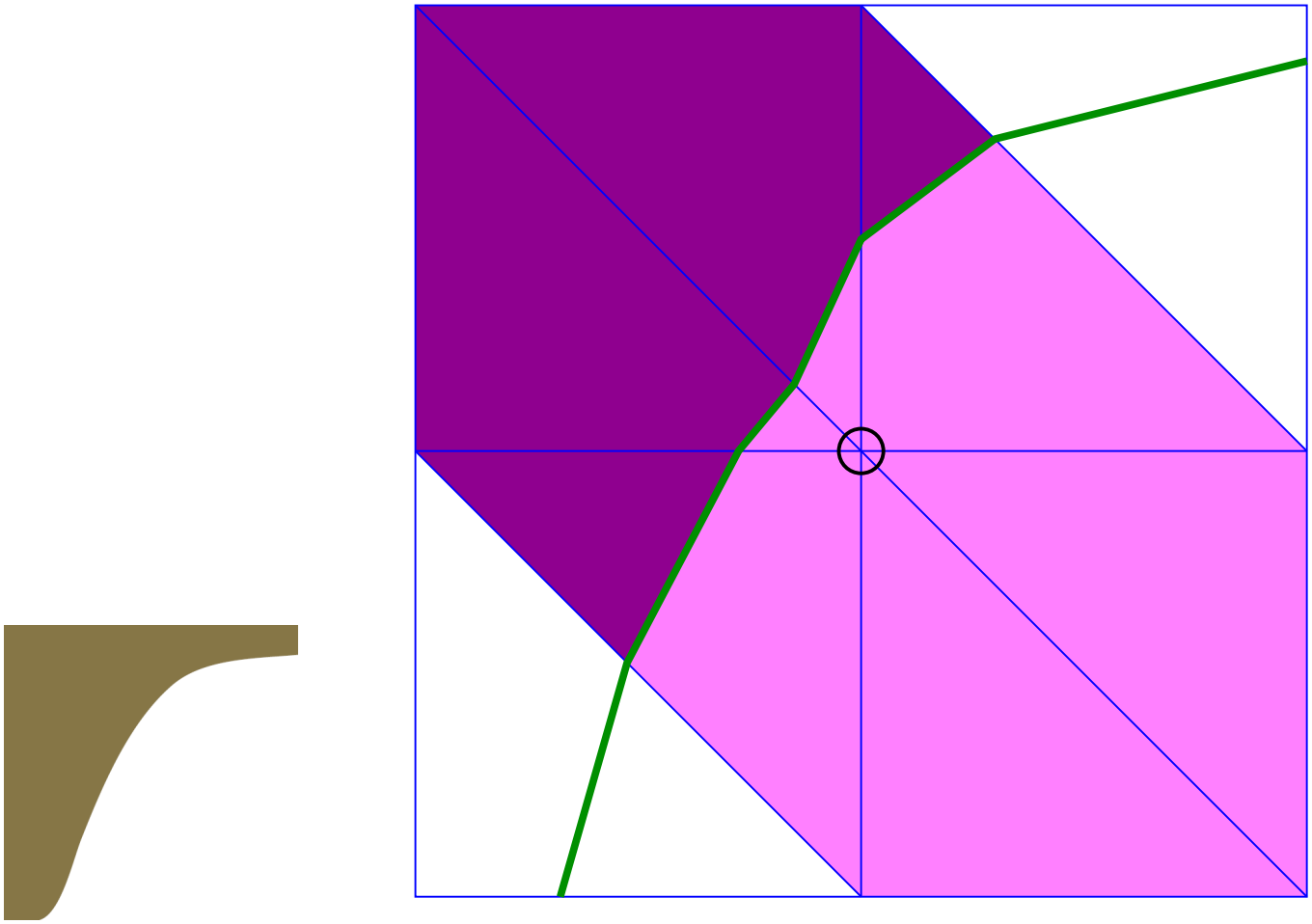

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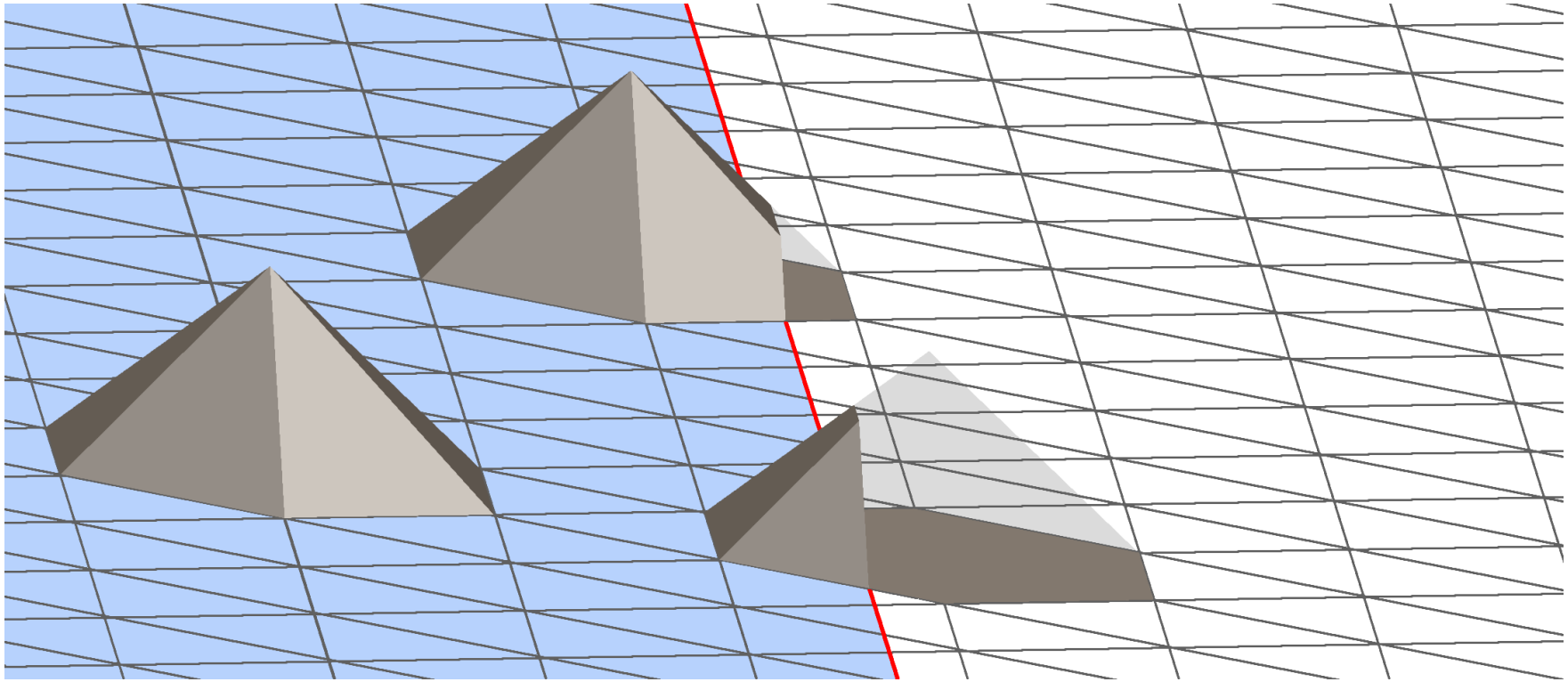

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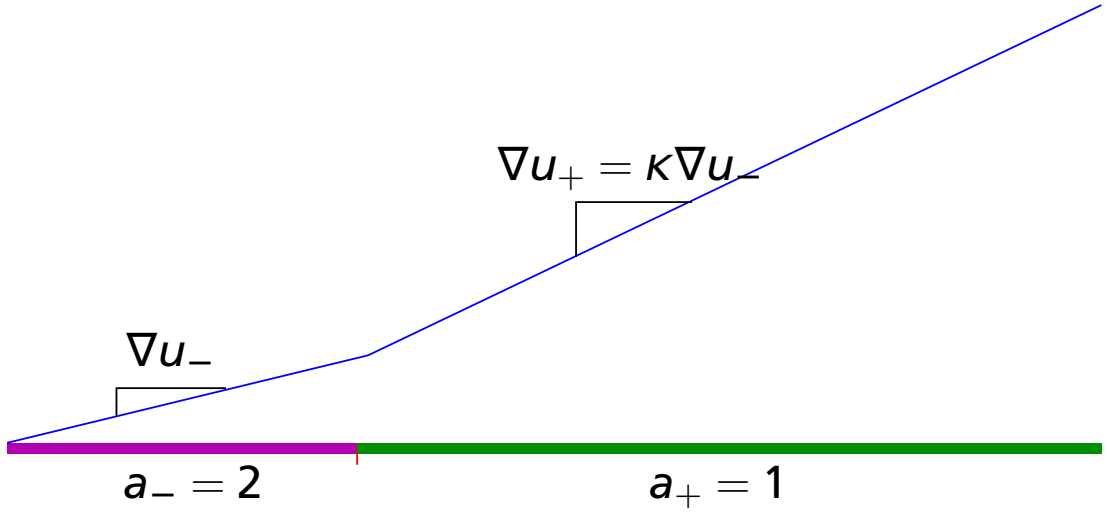
Discontinuous Coefficients (1D): Coupling Conditions

Heat diffusion (steady state)

$$-\operatorname{div}(a\nabla u) = f$$

with discontinuous coefficient a.



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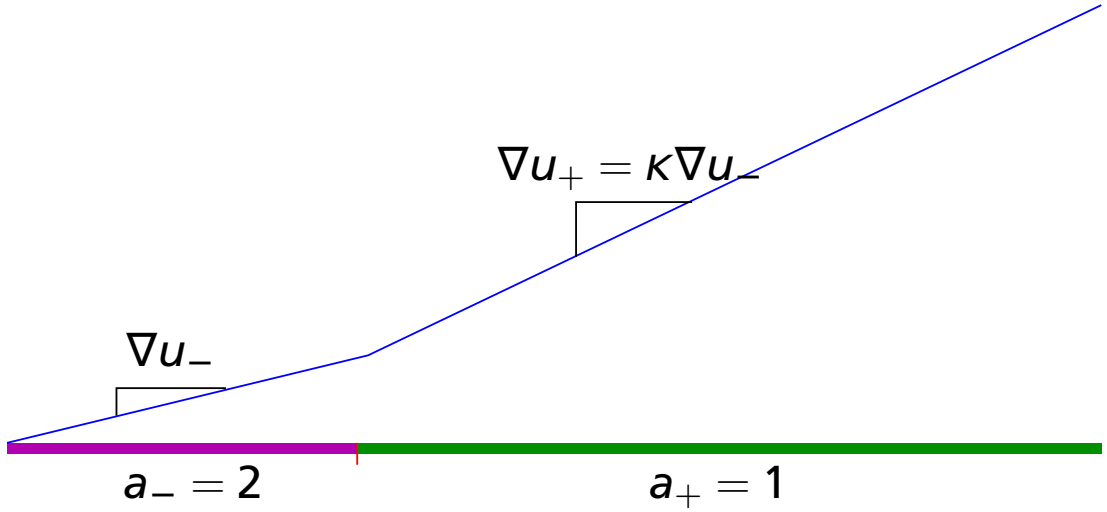


Kink ratio ("slope outside over slope inside"):

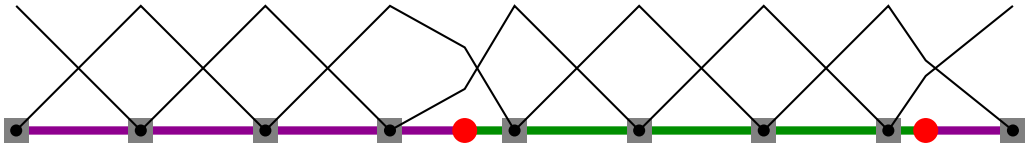
$$a\nabla u \text{ continuous} \quad \Rightarrow \quad \frac{a_-}{a_+} = \frac{\nabla u_+}{\nabla u_-} =: \kappa$$

2. Composite Finite Elements

CFE for Discontinuous Coefficients (1D)

3D: [Preusser et al., SIAM J Sci Comput 2011]

- CFE basis functions adapted to interpolate functions satisfying said coupling condition



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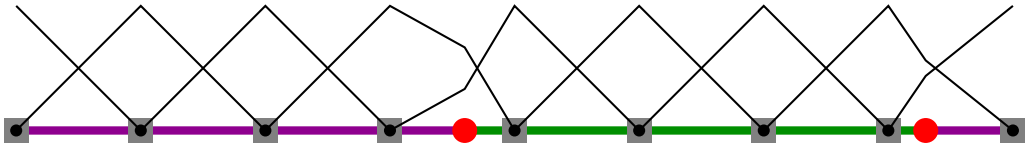
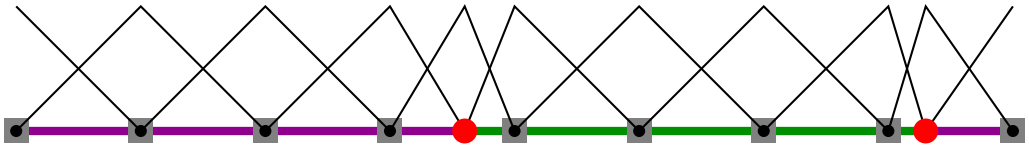


- constructed from standard basis on ‘local auxiliary mesh’



used for composing only

2. Composite Finite Elements

CFE for Geometrically Complicated Domains (2D)

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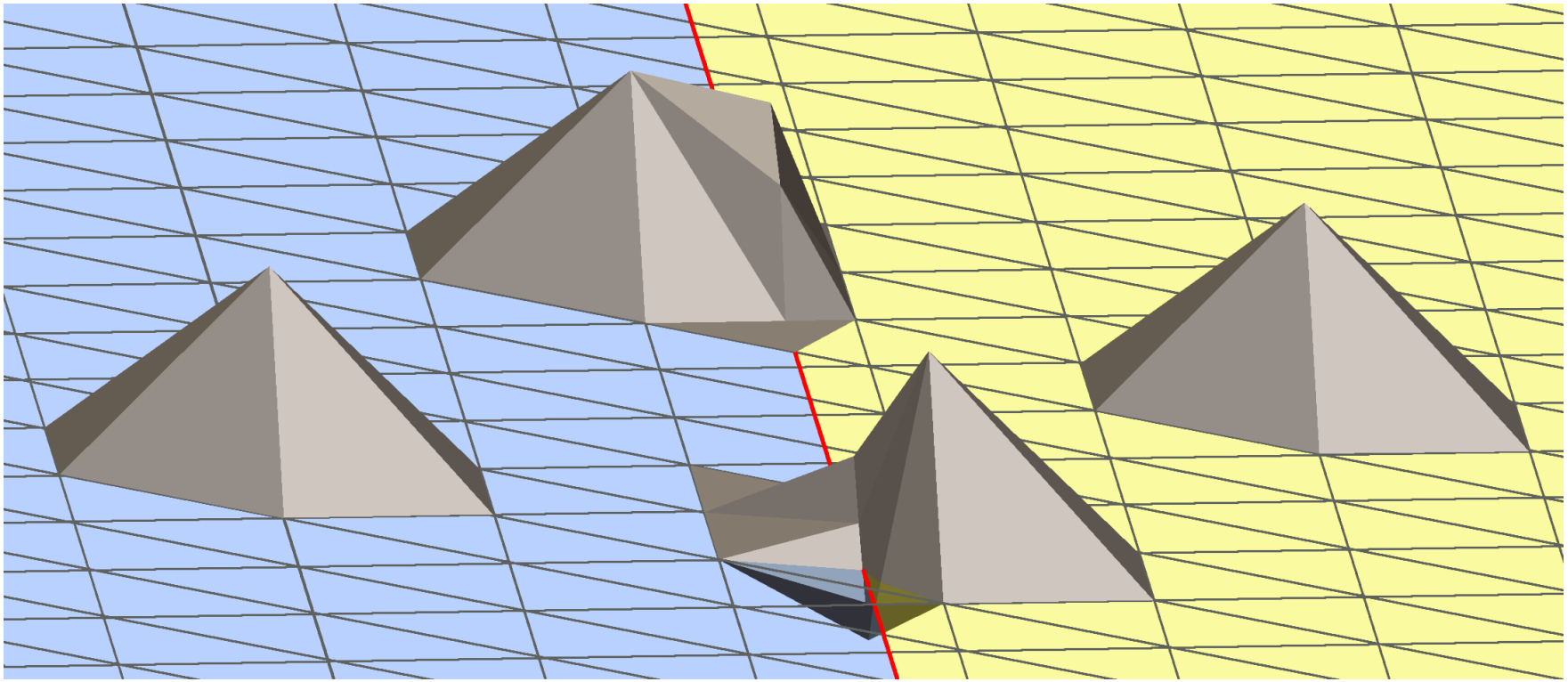

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- possibly negative, possibly locally extended support

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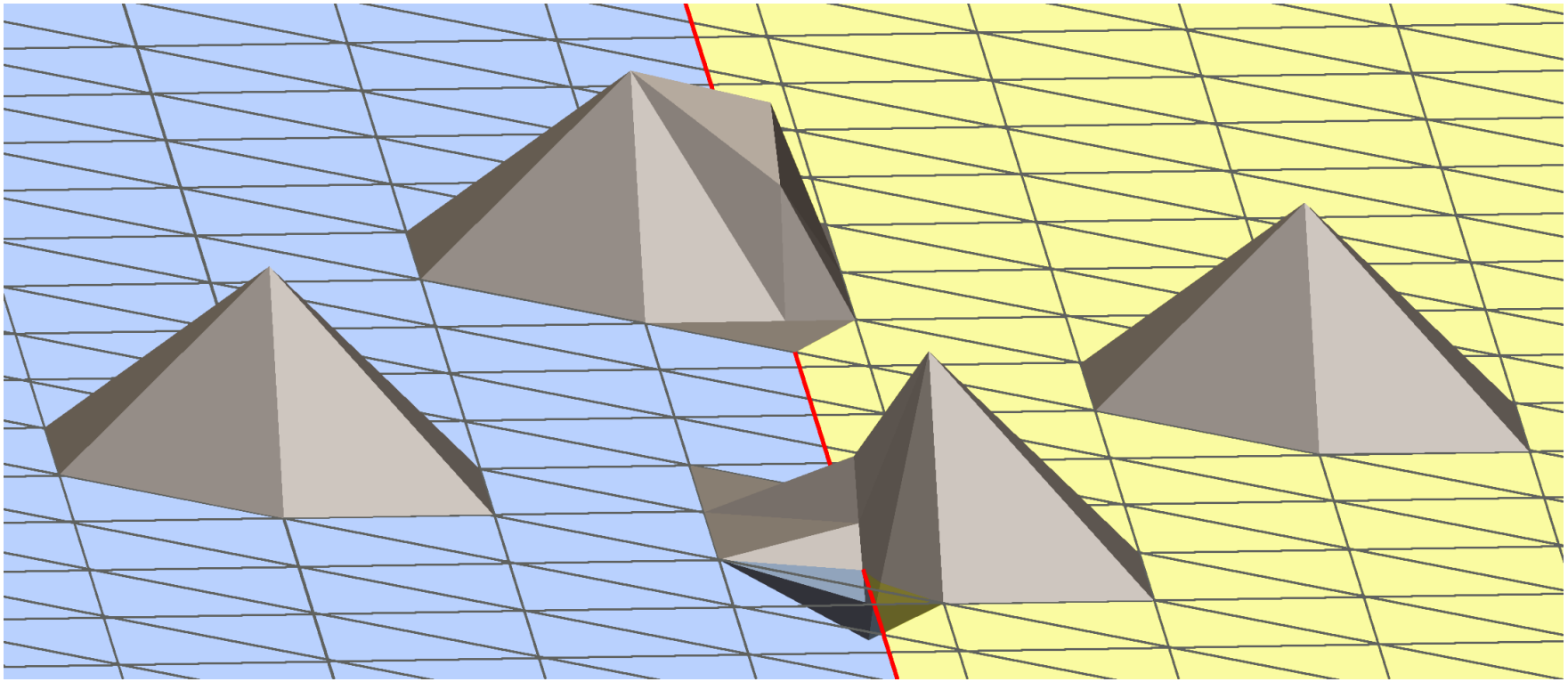

- partition of unity
- possibly negative, possibly locally extended support
- actual implementation in 3D

2. Composite Finite Elements

CFE Further Details

[S. et al, ECCOMAS 2012]

- elasticity problem (vector-valued)

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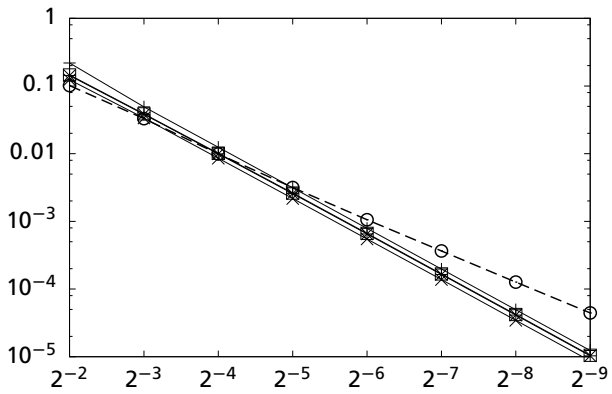
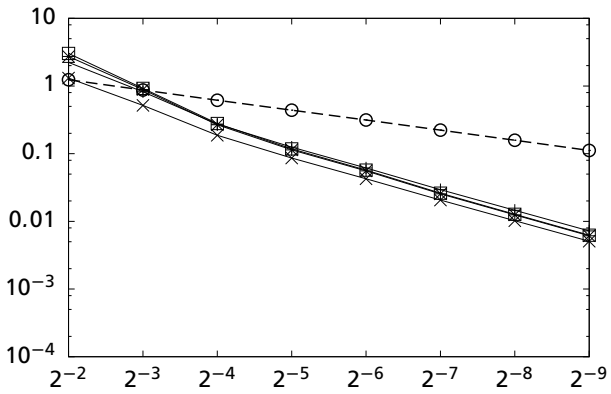
- combination of complicated domain and discontinuous coefficients
- numerically observe expected convergence order

L^2 error vs. grid spacing



H^1 error vs. grid spacing



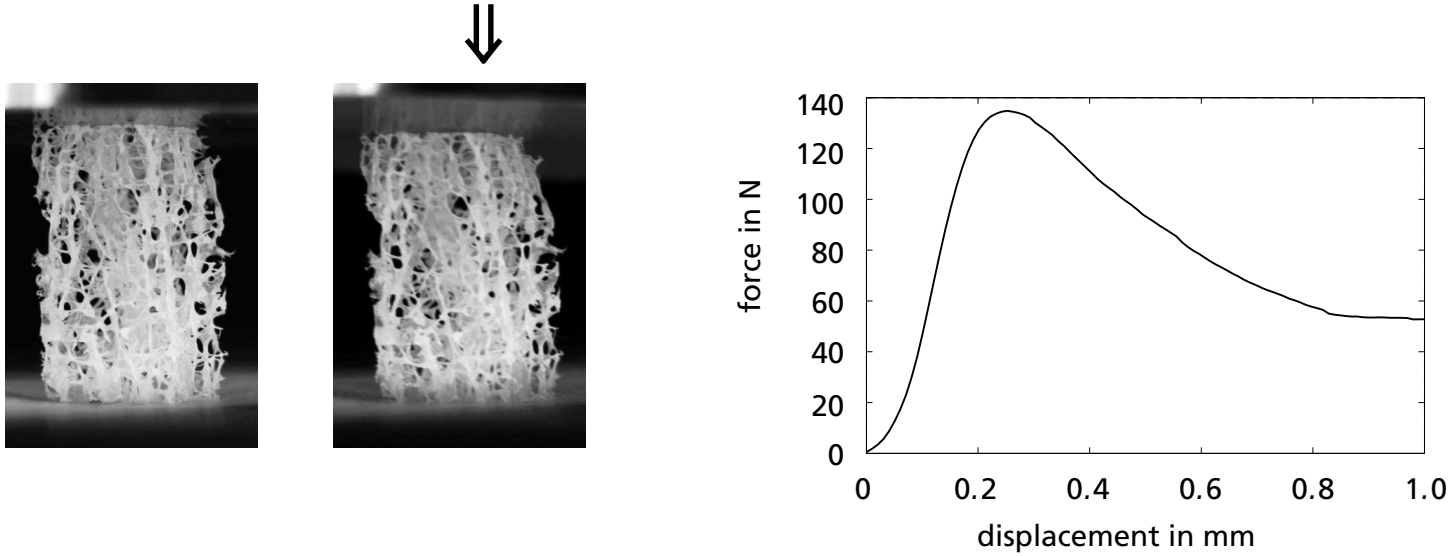
CFE: $\kappa = 1/16$ —+—, 4 —×—, 64 —✳—, 1024 —□—
stdFE, $\kappa = 4$ -⊙-

2. Composite Finite Elements

Experimental Validation

[S., Wolfram, CMBBE 2012]

- trabecular bone specimens: complicated domain
- linear elasticity



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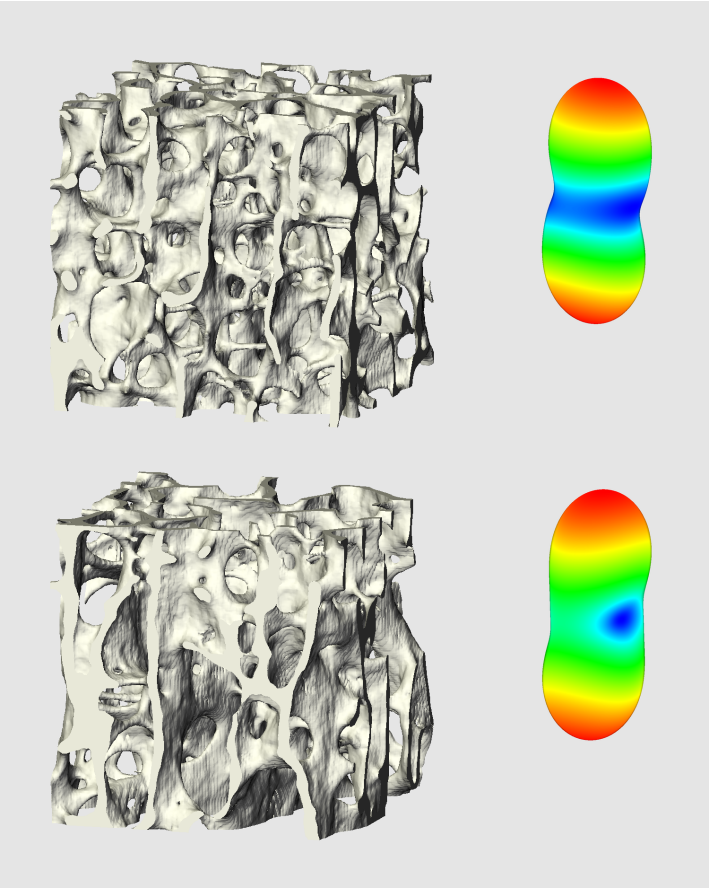
Elasticity of Trabecular Bone

- trabecular bone in vertebrae
- osteoporosis is degradation and loss of stiffness



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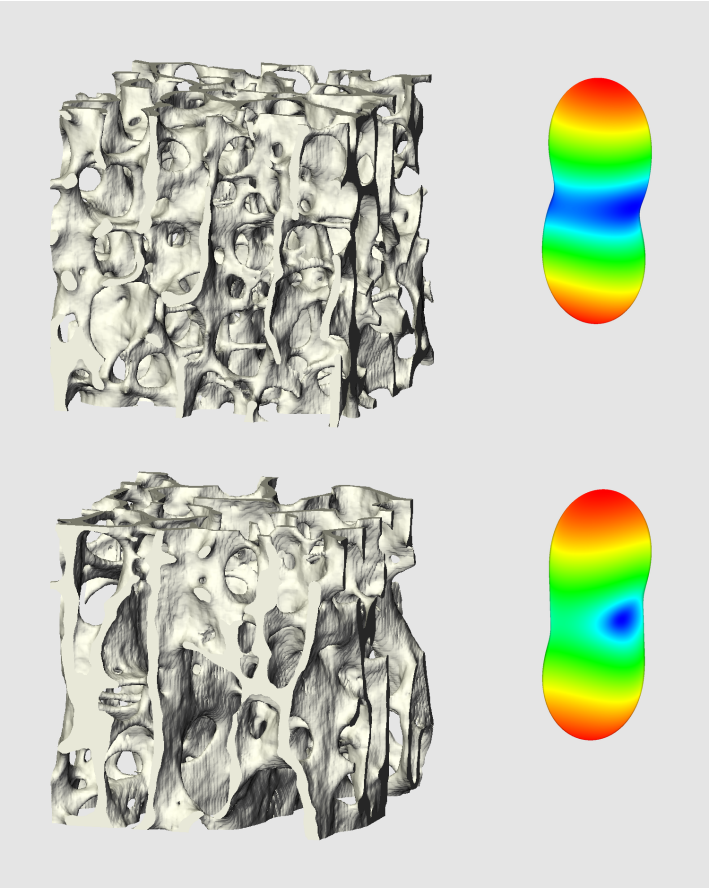

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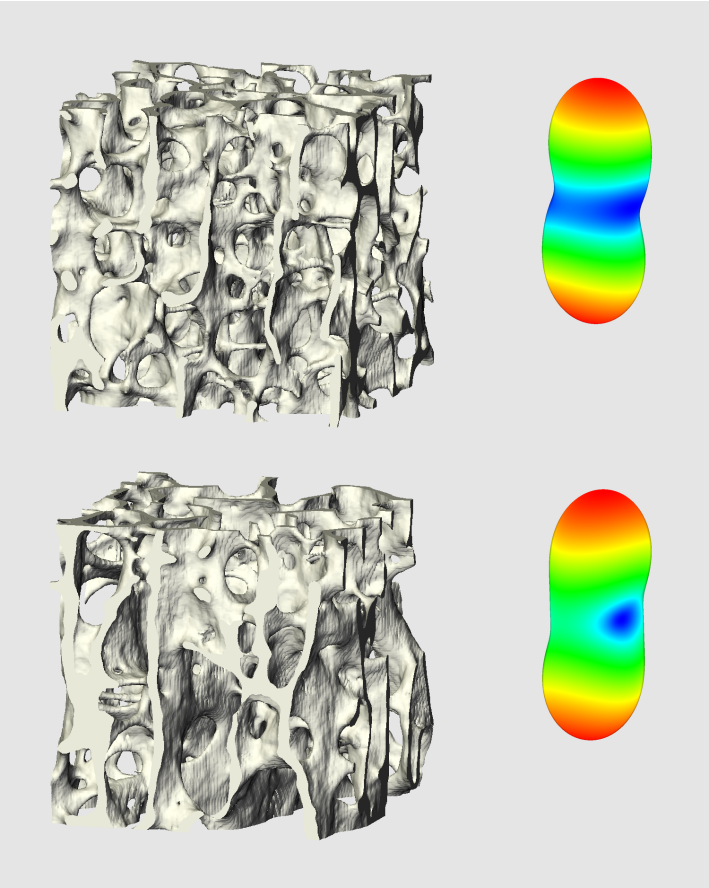

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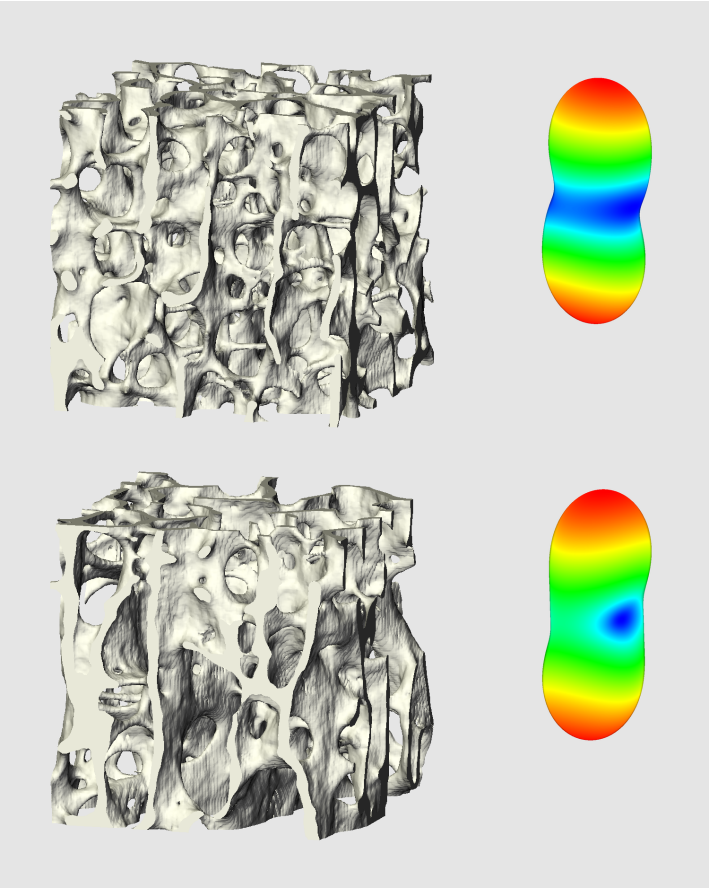
- “peanut” visualization



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Two-Scale Simulation: Fine Scale

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- **CFE (complicated domain)** elasticity simulations on cubic subspecimens

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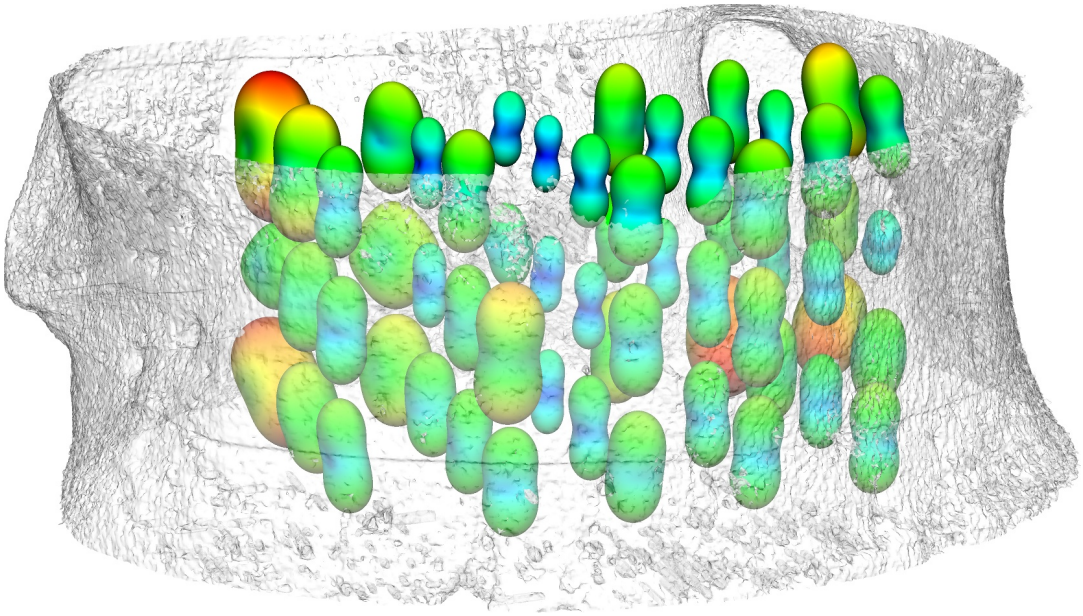

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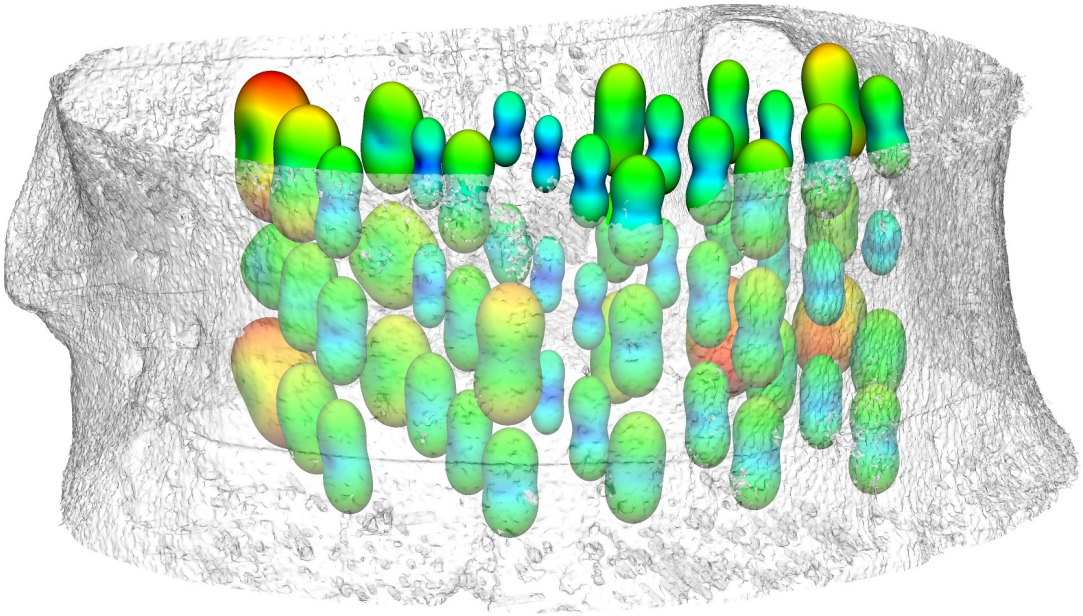

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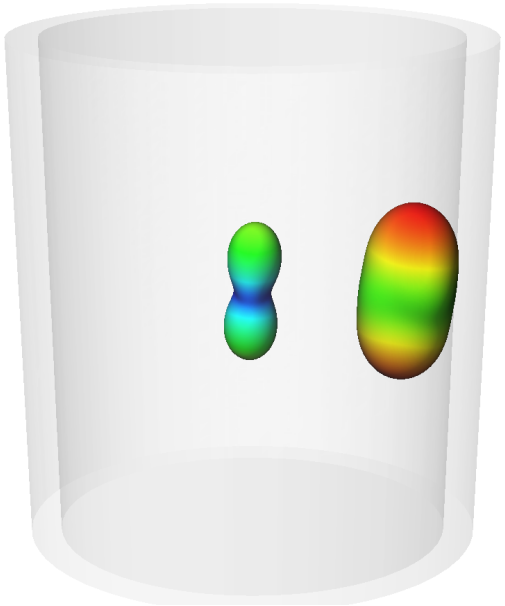

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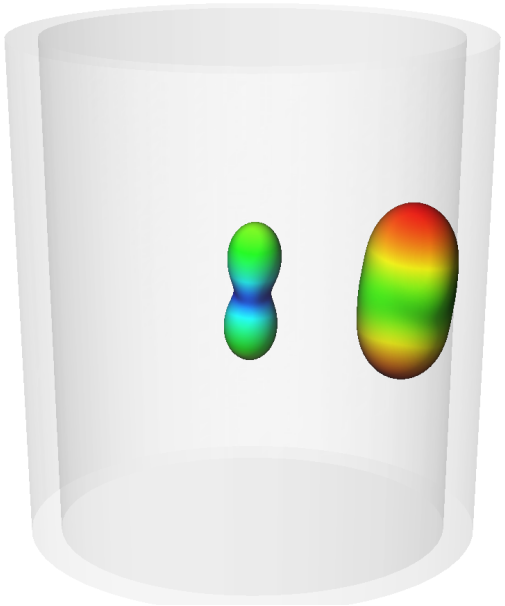

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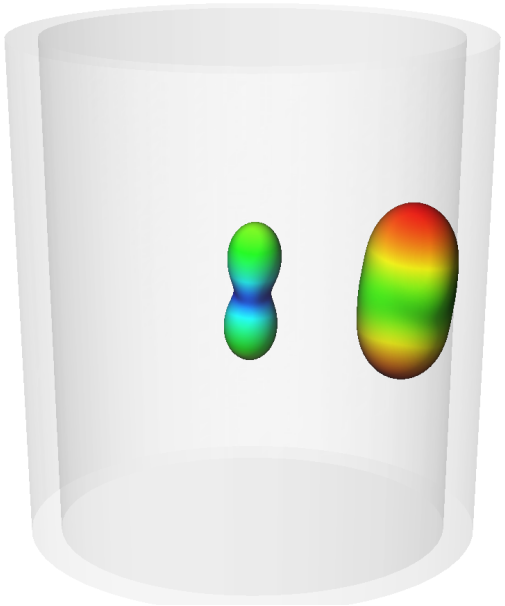

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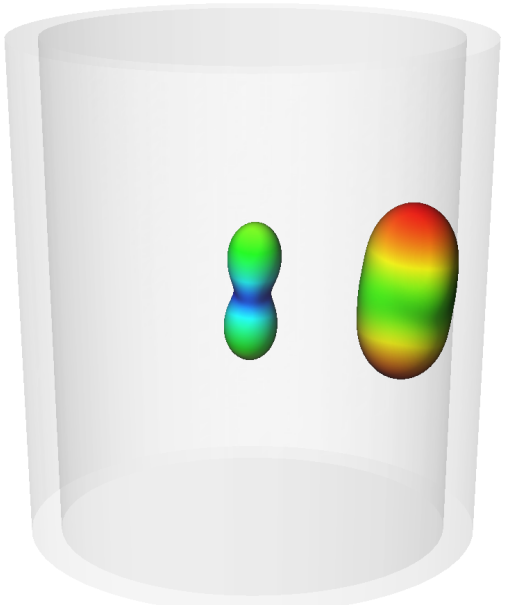

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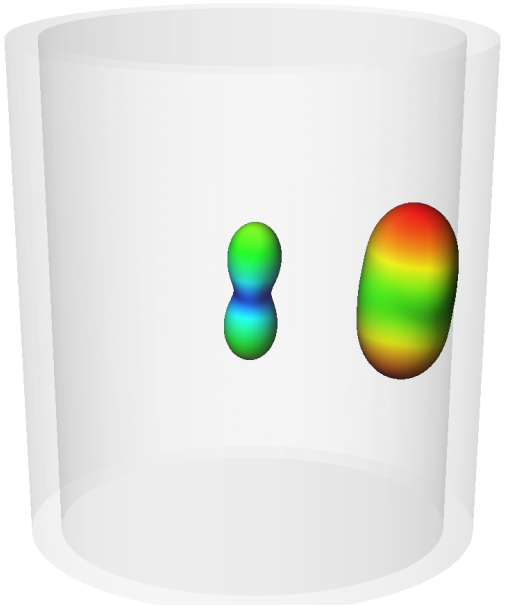

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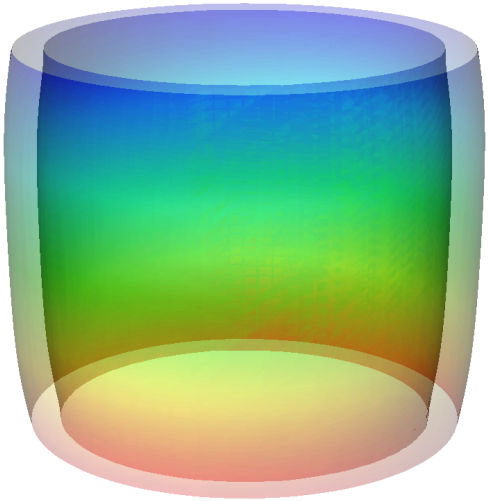

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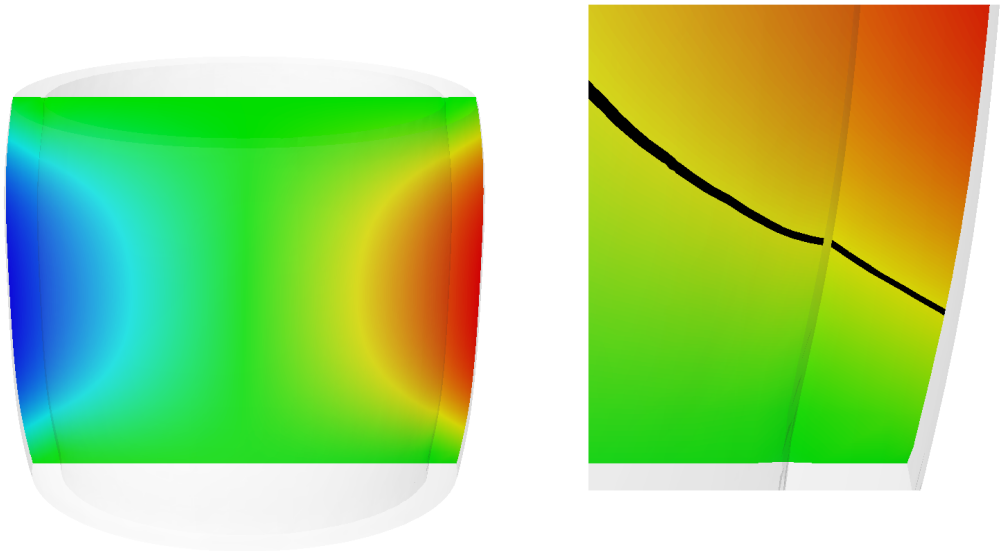

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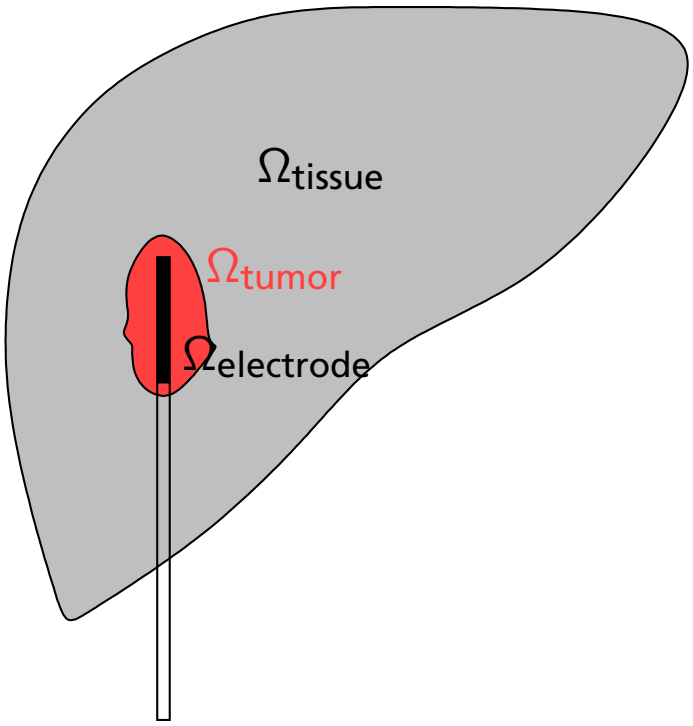
Radio Frequency Ablation

- RF probe inserted to/near tumor
- heating to destroy tumor
- while preserving surrounding tissue



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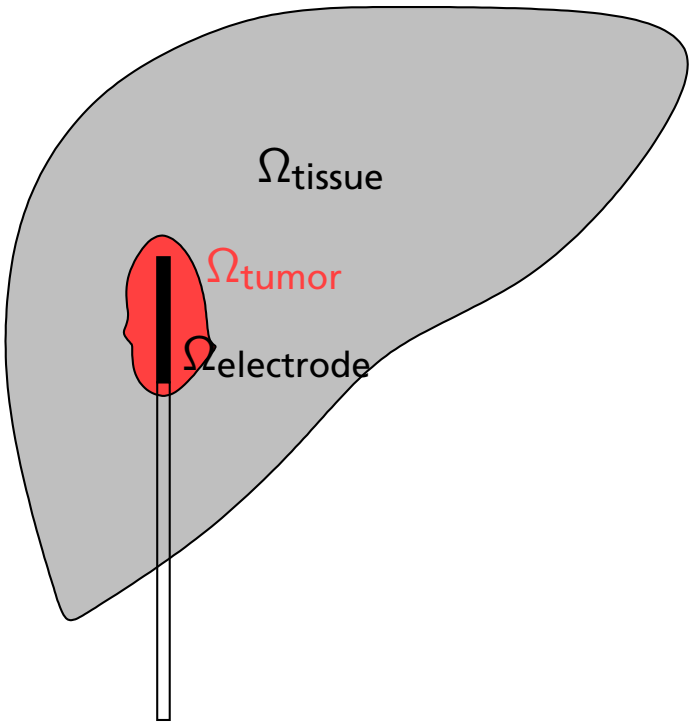

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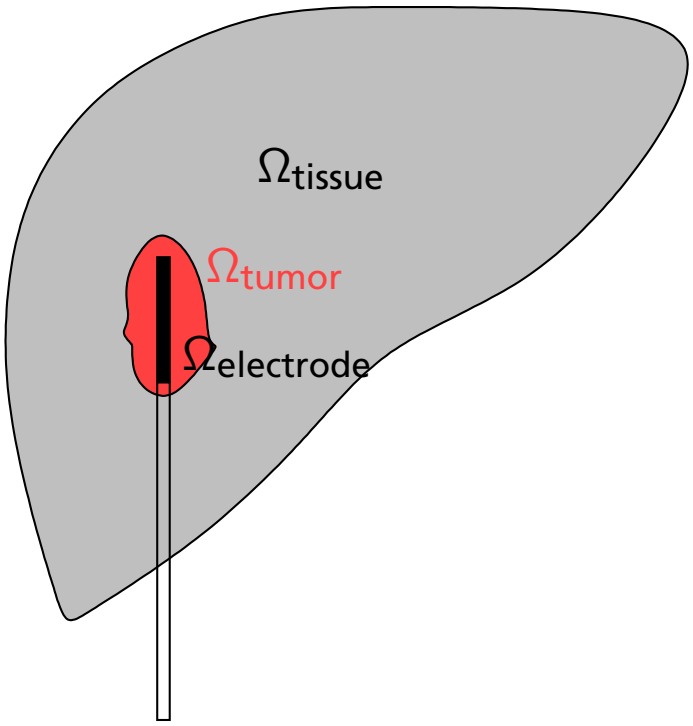

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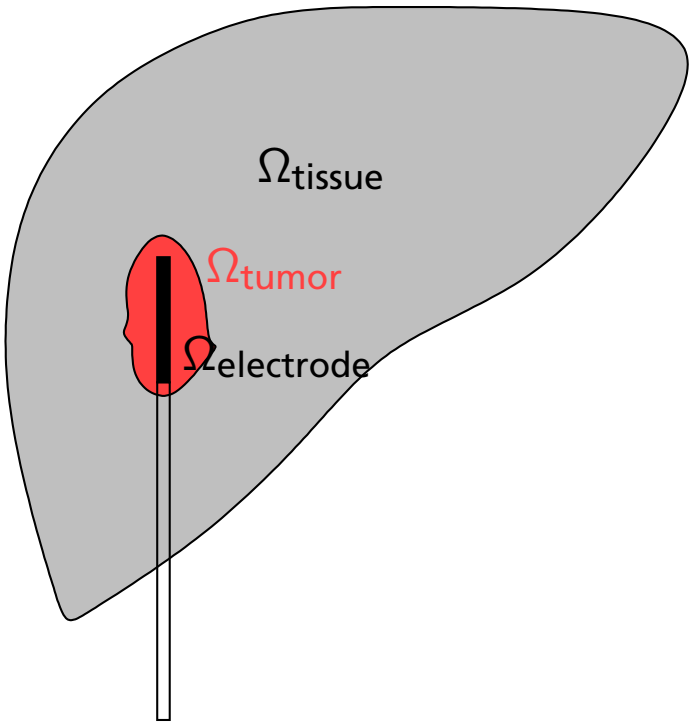
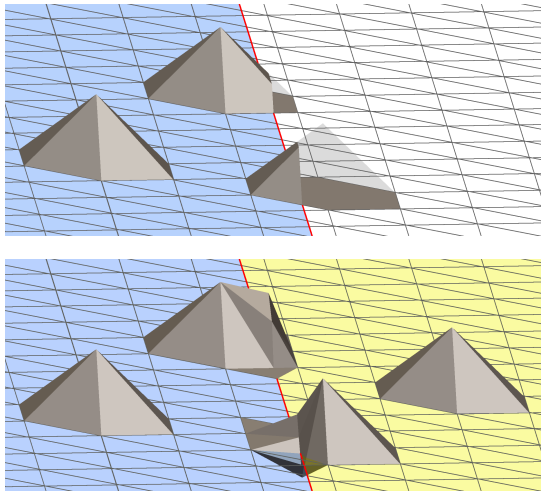
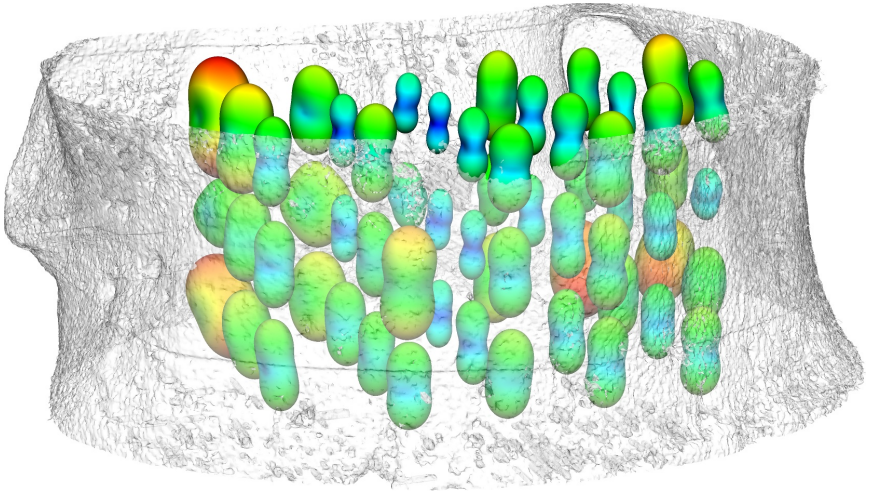
- discontinuous coefficient across $\overline{\Omega_{\text{tumor}}} \cap \overline{\Omega_{\text{tissue}}}$



Summary





- CFE for complicated domains and discontinuous coefficients
- biomedical applications

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CFE implementation available open source from
http://numod.ins.uni-bonn.de/software/quocmesh/index.html

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