CONSTRUCTIVE ALGORITHMS FOR MODELING REALISTIC VASCULAR STRUCTURES

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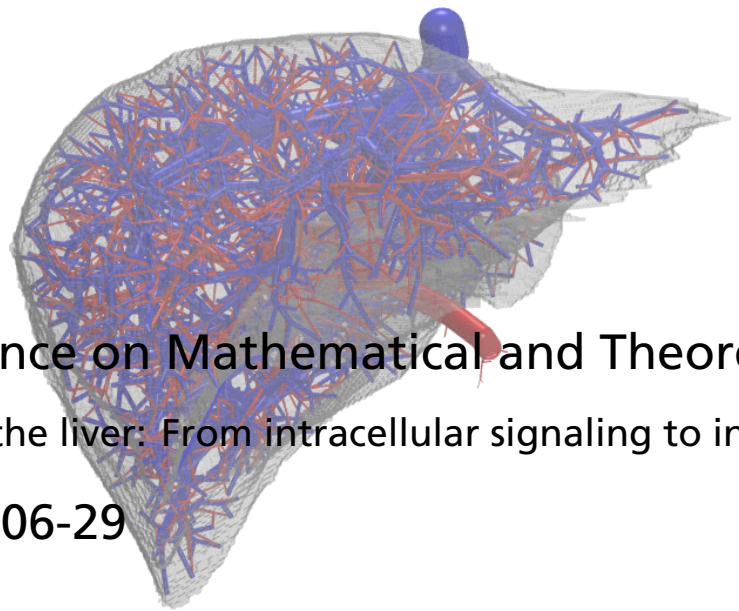
Joint work with Karolis Blaževičius, Tobias Preusser
Fraunhofer MEVIS and Jacobs University, Bremen



8th European Conference on Mathematical and Theoretical Biology

Multi-scale mathematics of the liver: From intracellular signaling to intercellular interaction

ECMTB Kraków, 2011-06-29

virtual liver network

Fraunhofer MEVIS

0.
Outline

1. Introduction

2. Geometric Analysis of Vascular Structures

3. Statistical Comparison of Geometric Features

4. Algorithmic Generation of Vascular Structures

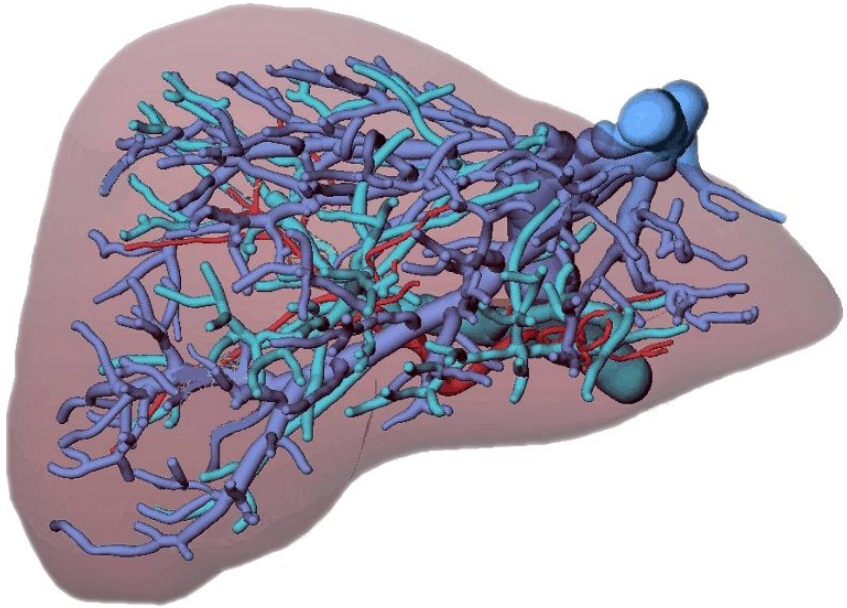
5. Outlook

1. Introduction
Contents

1. Introduction

The Liver

- organ with impressive regeneration capabilities
- connected to organism by four vascular systems



  - hepatic artery (HA) || portal vein (PV) for supply
  - hepatic vein (HV) for drainage
  - bile duct (not shown)
- lobuli (⌀ ≈ 1 mm) as functional units

1. Introduction
Imaging Vascular Systems

- in vivo 3D imaging
- corrosion casts and µCT (3D)

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- in vivo 3D imaging
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- gap between organ and lobular scale
- 2D microscopy / $(2+\epsilon)$ D histological slices

1. Introduction

Generating Vascular Systems

Goal: bridge the resolutional gap

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2. perform geometric analysis, evaluate measures of similarity (calibration)

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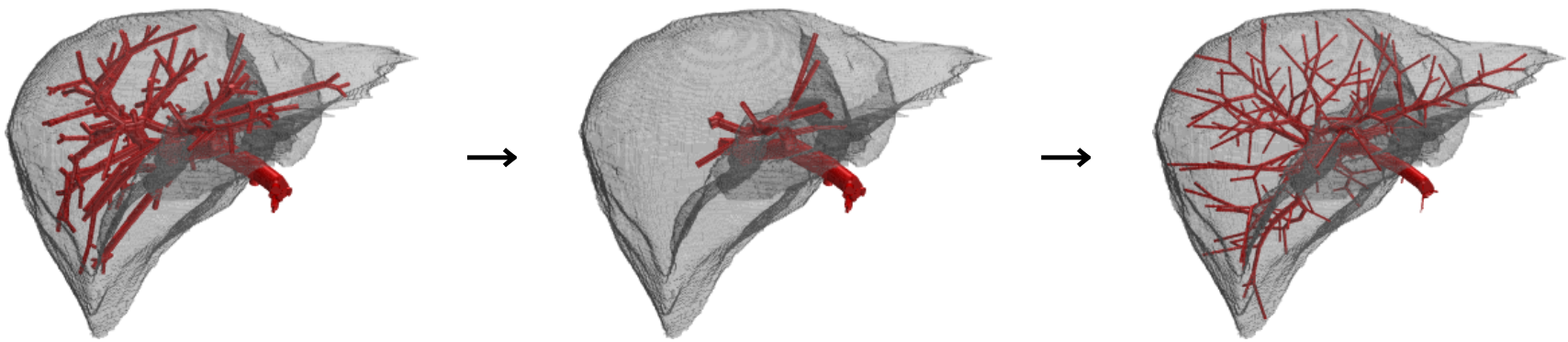
1. obtain geometric representation of real vascular tree structures (~150 clinical CT scans)
2. perform geometric analysis, evaluate measures of similarity (calibration)
3. implement algorithm for generating vascular trees
4. starting from very coarse trees, validate semi-fine generated against measured trees (validation)



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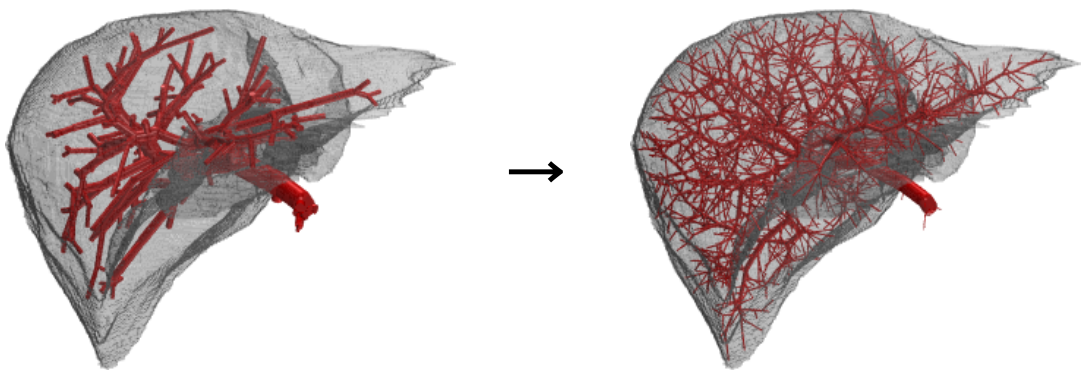
Generating Vascular Systems

Goal: bridge the resolutional gap

1. obtain geometric representation of real vascular tree structures (∼150 clinical CT scans)
2. perform geometric analysis, evaluate measures of similarity (calibration)
3. implement algorithm for generating vascular trees
4. starting from very coarse trees, validate semi-fine generated against measured trees (validation)
5. starting from measured trees, generate additional resolution (application)



2. Geometric Analysis of Vascular Structures
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Skeletonization of Vascular Trees

- 3D image data
- segmentation and skeletonization

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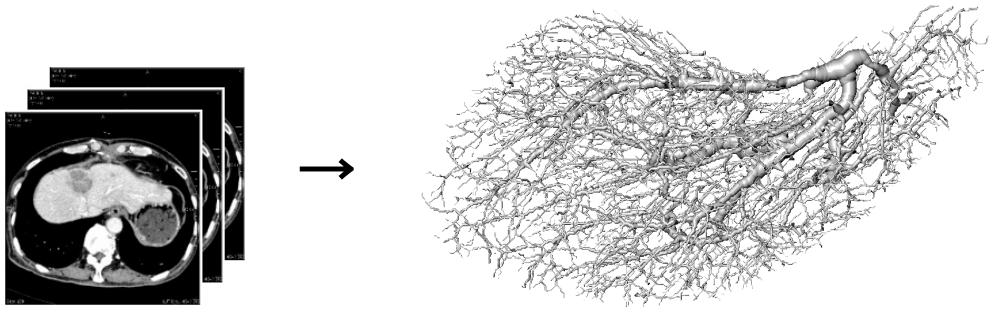
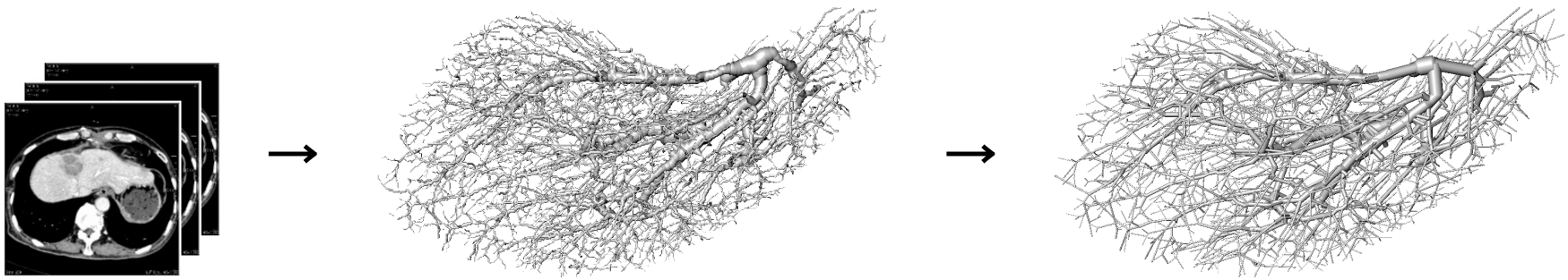


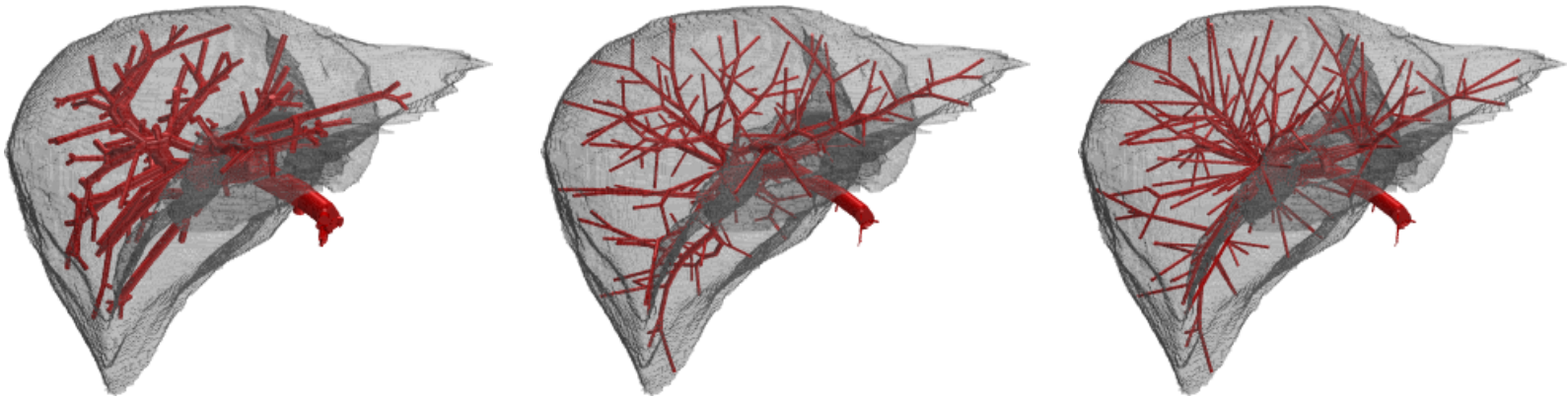
2. Geometric Analysis of Vascular Structures
Skeletonization of Vascular Trees

- 3D image data
- segmentation and skeletonization
- graph representation of different hepatic vascular networks
- simplifications
  - strictly bifurcative
  - straight cylinder segments (constant radius, not curved)



2. Geometric Analysis of Vascular Structures

Comparison of Vascular Trees



- visual comparison is only qualitative, but important
- need systematic appropriate comparison of geometric features

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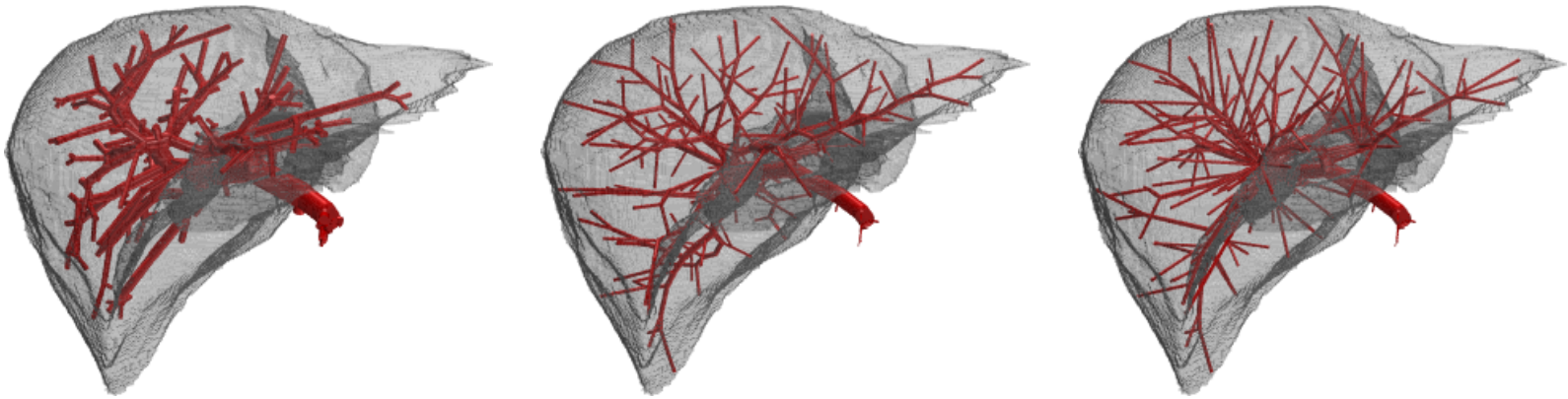

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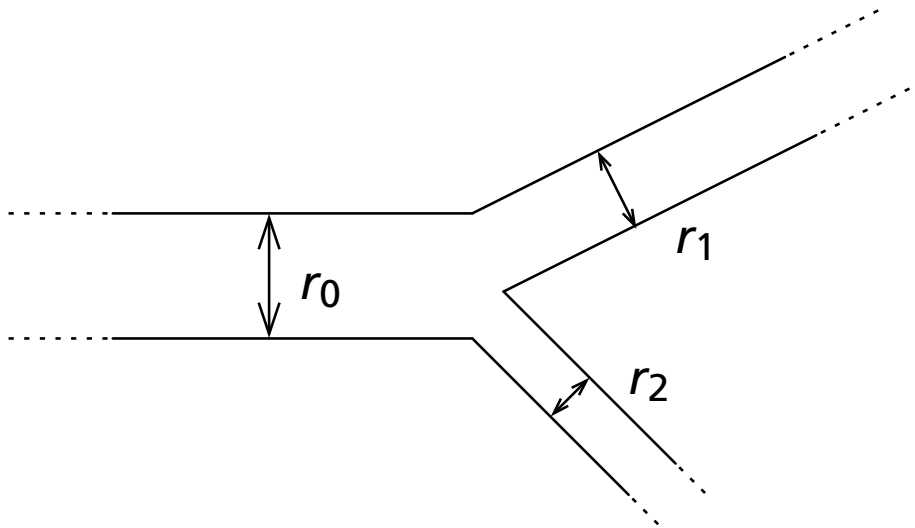
Geometric features include …

2. Geometric Analysis of Vascular Structures
Geometric Features (Radii)

- radii r_0
- expansion factors $\frac{r_1}{r_0}$, $\frac{r_2}{r_0}$
- asymmetry $\frac{r_2}{r_1}$
- 'bifurcation exponent' γ satisfying $r_0^\gamma = r_1^\gamma + r_2^\gamma$



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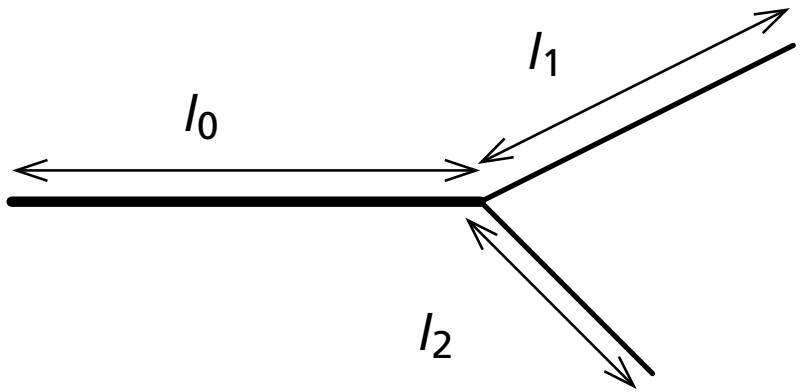
Geometric Features (Lengths)

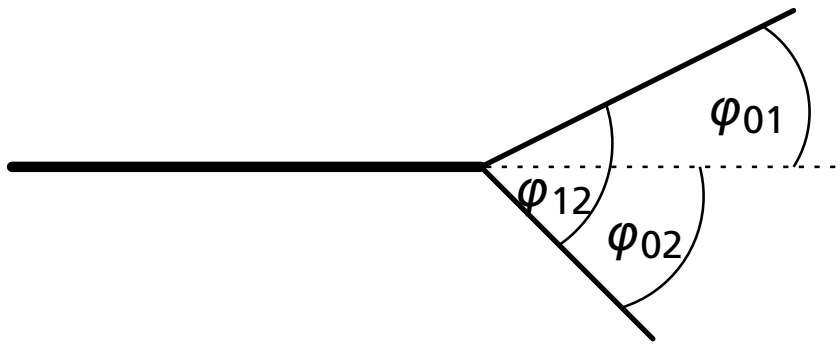
- lengths l_0
- expansion factors $\frac{l_1}{l_0}$, $\frac{l_2}{l_0}$
- asymmetry $\frac{l_2}{l_1}$



2. Geometric Analysis of Vascular Structures

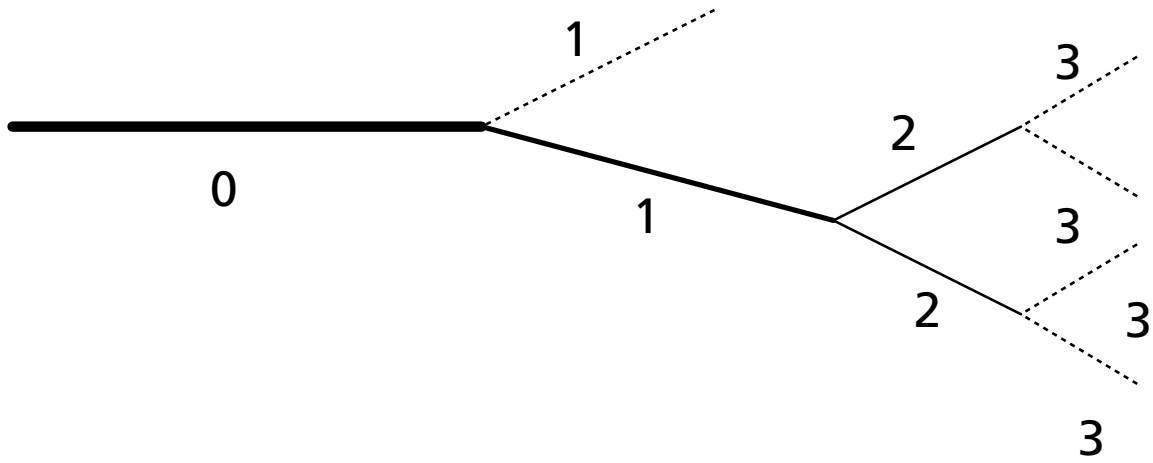
Geometric Features (Angles in 3D)

- between parent and daughters φ_{01}, φ_{02}
- between daughters φ_{12}
- $\varphi_{01} + \varphi_{02} (\neq \varphi_{12})$
- inclinations



2. Geometric Analysis of Vascular Structures
Strahler Orders

- bifurcation generations (from root to leaves) not a useful concept



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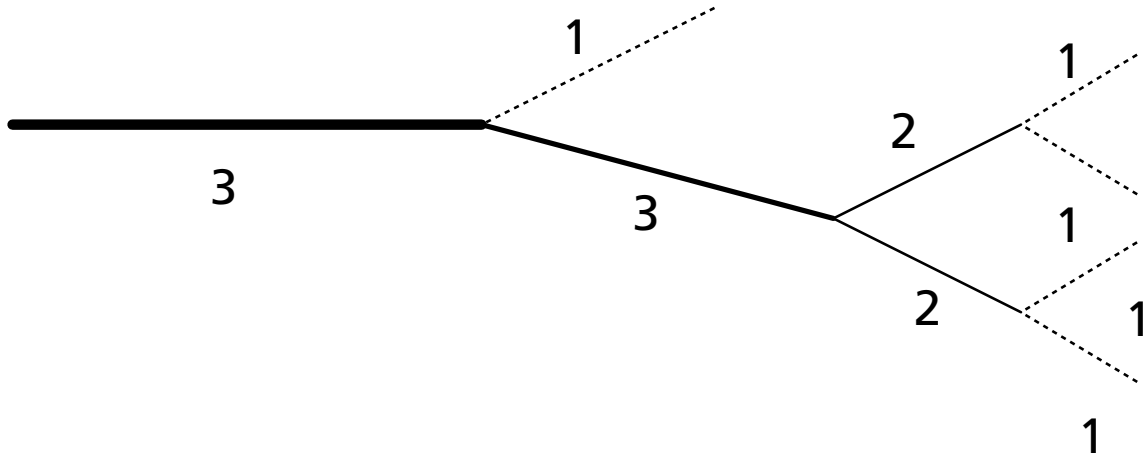

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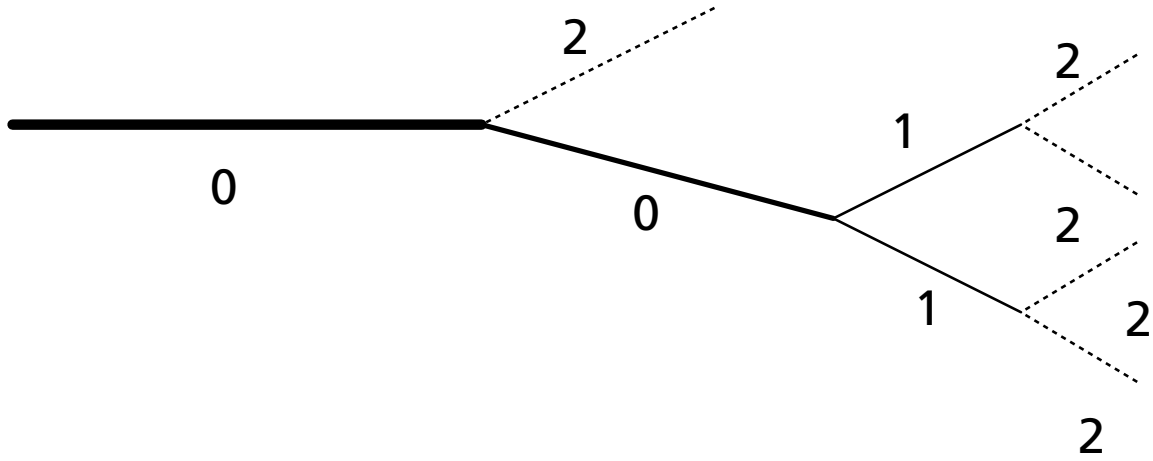

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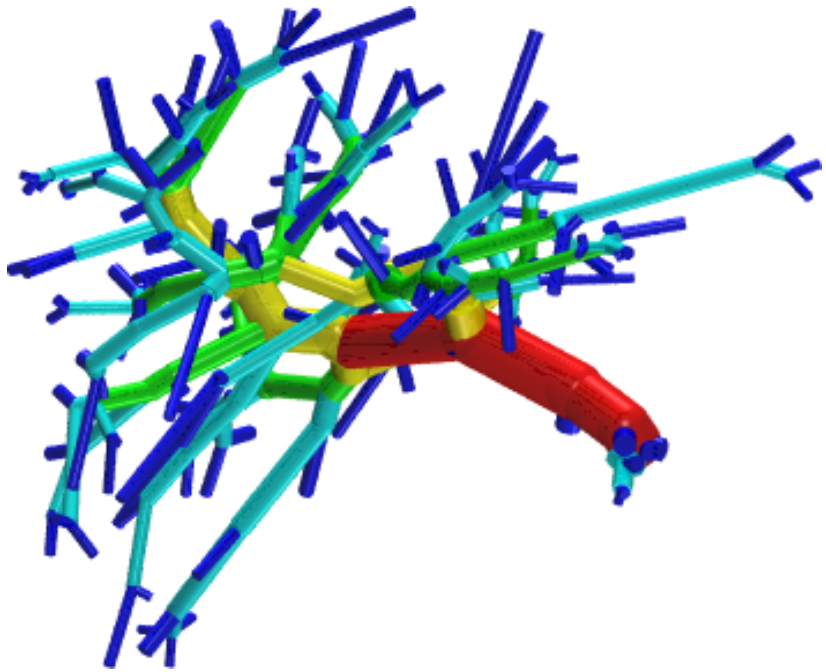

3. Statistical Comparison of Geometric Features
Contents

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Averaging

Consider different

- number of geometric features #geom
- number of Strahler* orders #Str*
- number of trees #Tr

Let us start with
(#geom = 1 | #Str* = 1 | #Tr = 1)

3. Statistical Comparison of Geometric Features
Statistical Interpretation

For a single tree T $(\#\text{geom} = 1 \mid \#\text{Str*} = 1 \mid \#\text{Tr} = 1)$

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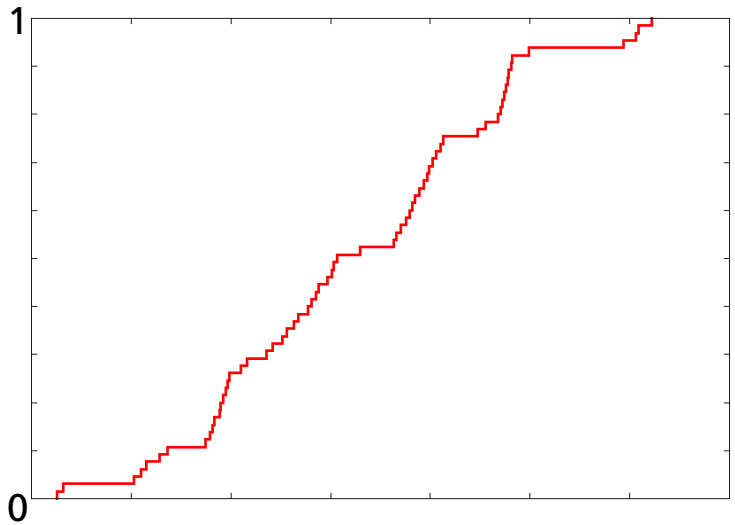
For a single tree T $(\#\text{geom} = 1 \mid \#\text{Str*} = 1 \mid \#\text{Tr} = 1)$

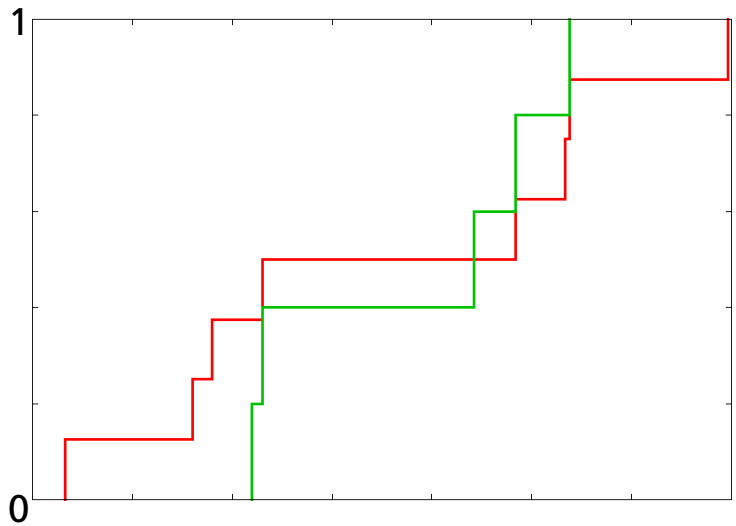
- compute geometric feature of interest
- determine Strahler* orders for each edge
- histogram per Strahler* order
  $\rightsquigarrow$ (probability) density $\rightsquigarrow$ cumulative distribution function (CDF) F

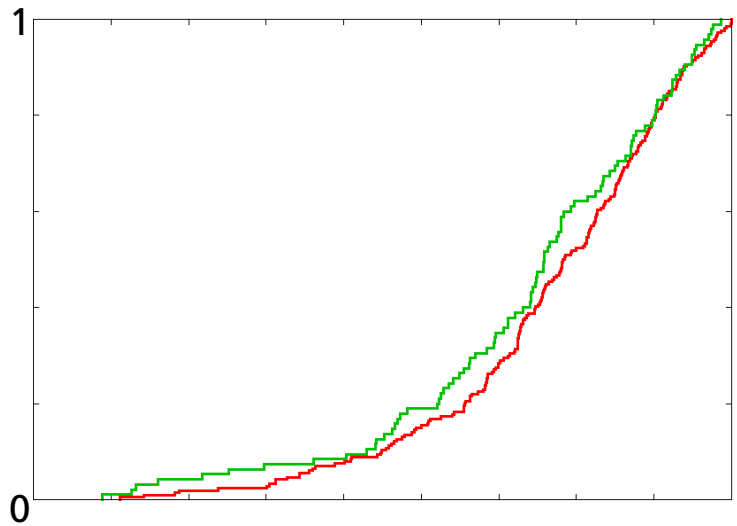


3. Statistical Comparison of Geometric Features

Comparison of Histograms

T_0, T_1 two trees with CDFs F_0, F_1 (#geom = 1 | #Str* = 1 | #Tr = 2)





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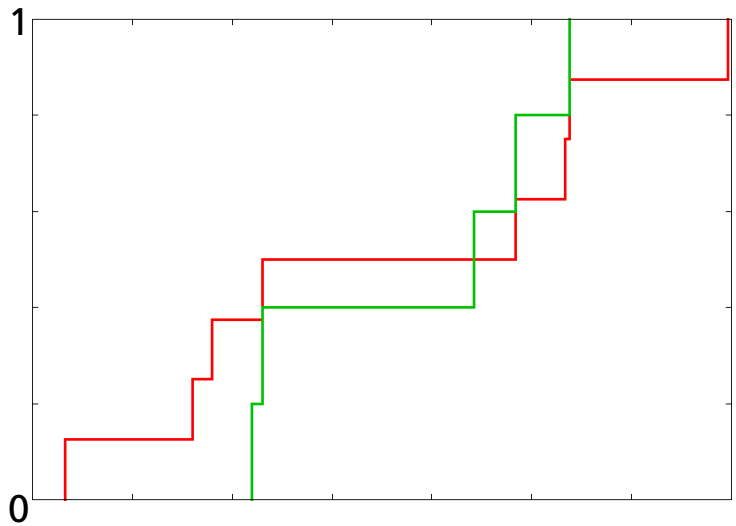
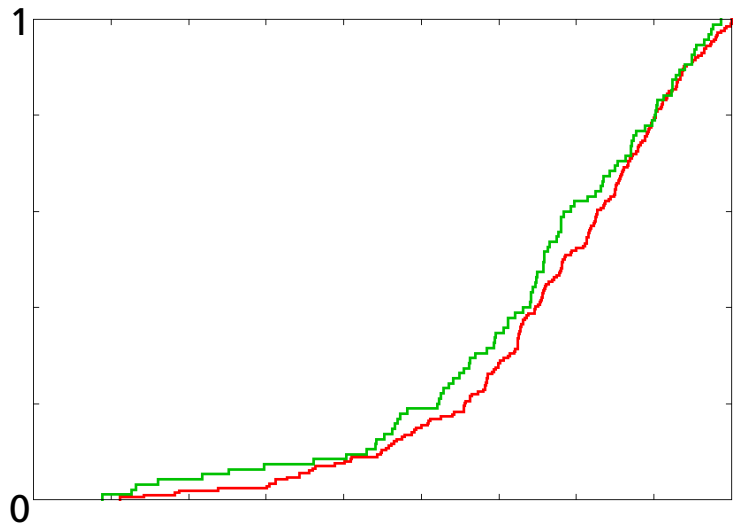
- $D_{KS} = \max |F_0 - F_1|$
  - independent of distribution (other than L^2)





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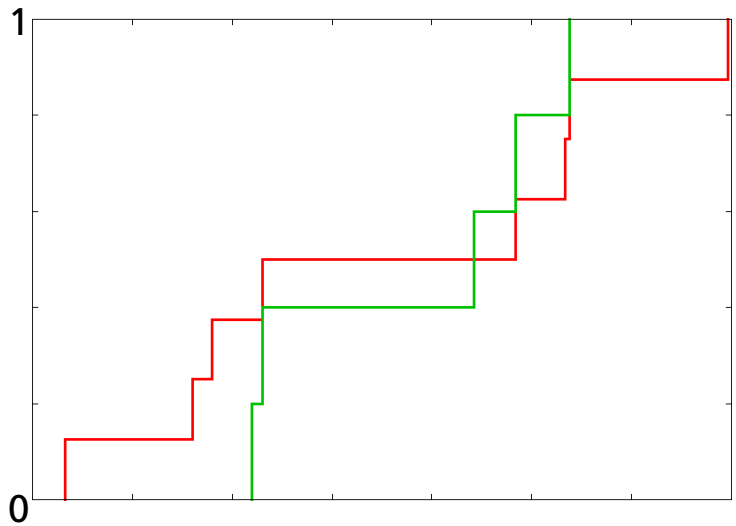
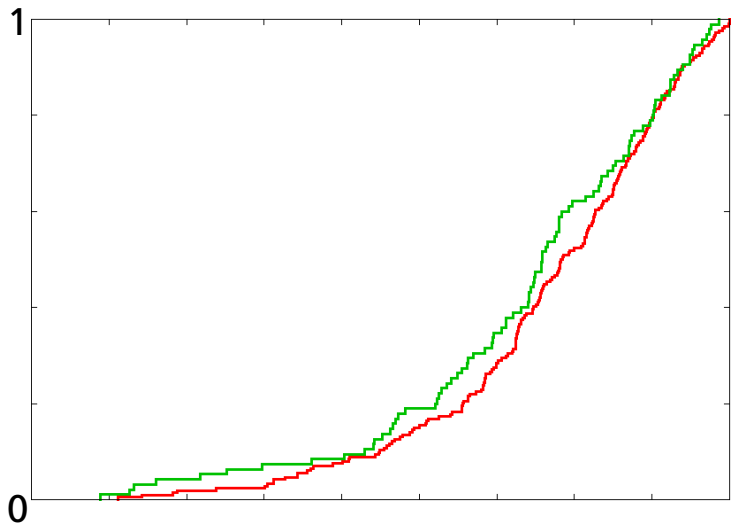


$P_{KS} = 0.780$



$P_{KS} = 0.692$

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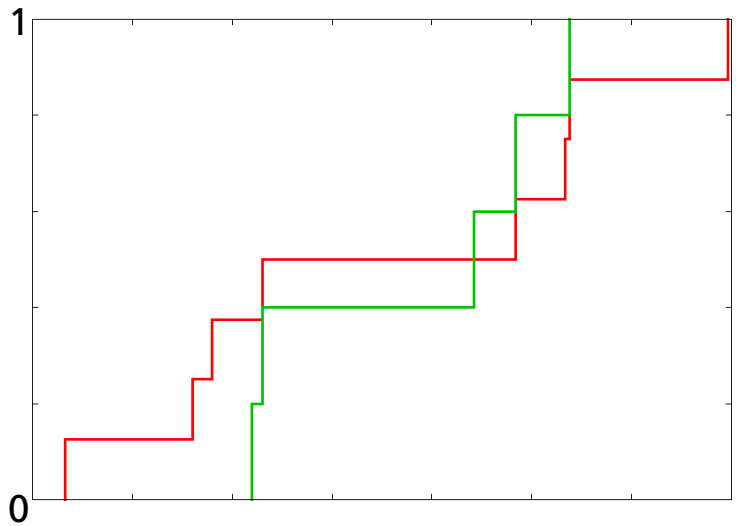
- $P_{KS}\left(\sqrt{\frac{n_0 n_1}{n_0+n_1}} D_{KS}\right)$ (Kolmogorov-Smirnov statistic; two-sample case)
  - F_0, F_1 drawn from the same distribution $\Rightarrow P_{KS} \in [0, 1]$
  - else $P_{KS} \to 0$ for increasing n_0, n_1
  - e.g. criterion $P_{KS} > 0.05 = 1 - \alpha$ for 'match'



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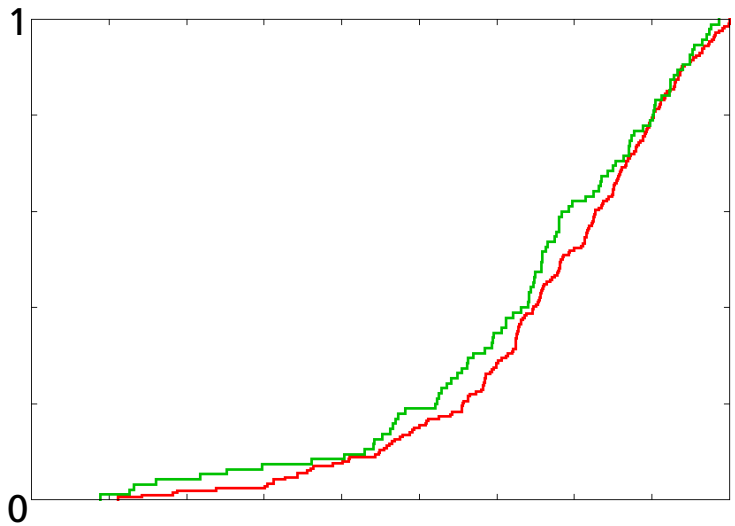

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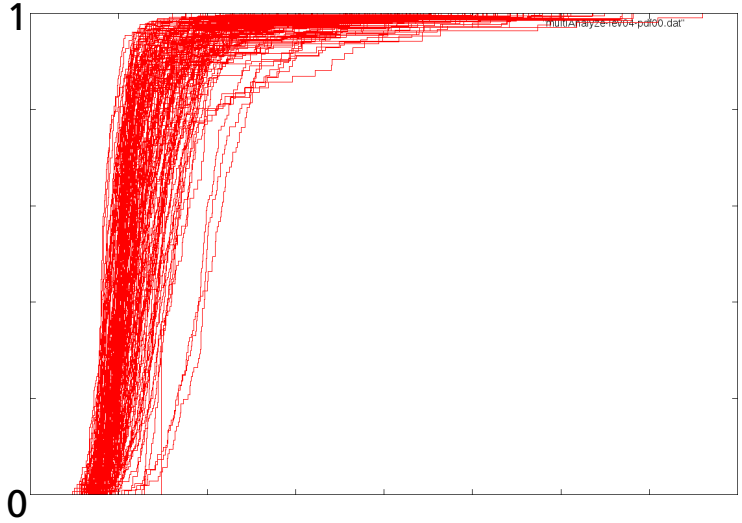
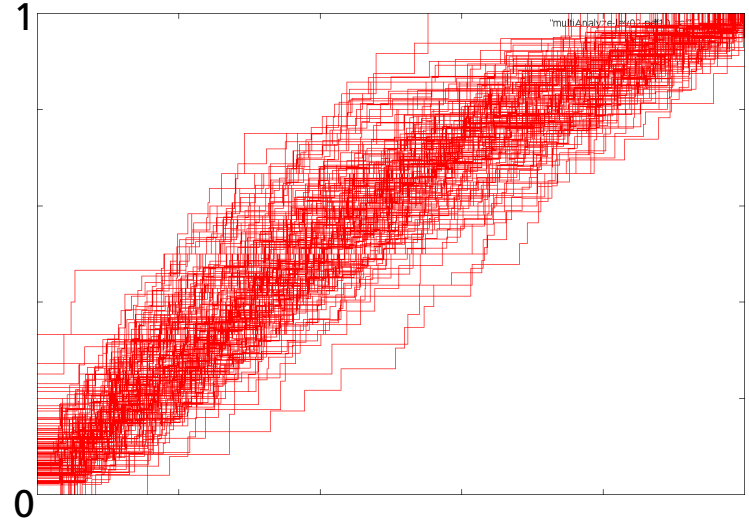
3. Statistical Comparison of Geometric Features

Comparison of Histograms: Match Ratios

$\mathcal{T} = \{T_0, \ldots, T_{N-1}\}$ set of trees with CDFs F_i, ($\#\text{geom} = 1 \mid \#\text{Str*} = 1 \mid \#\text{Tr} = N$)

- compute 'match ratio'





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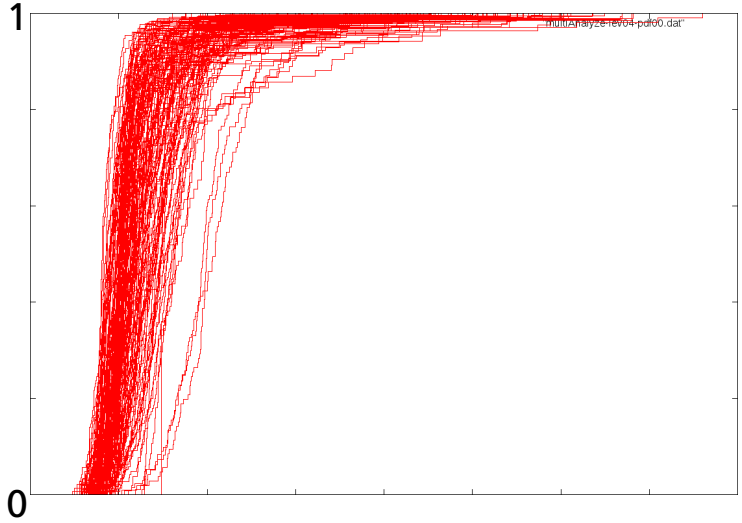
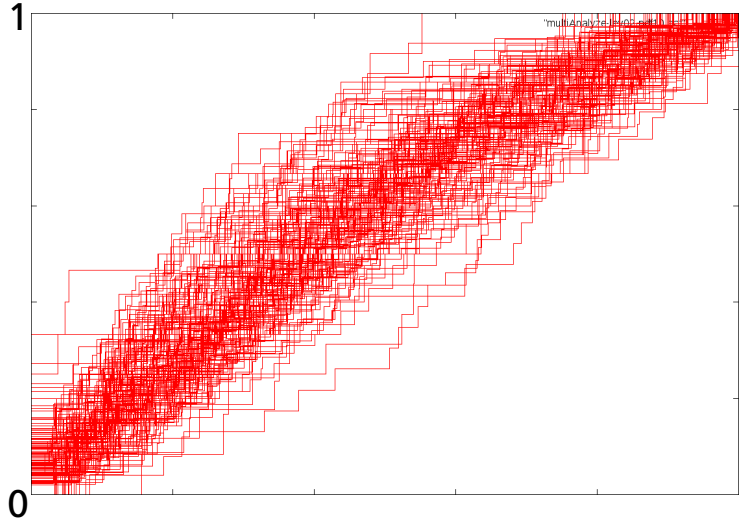
$$m_{\mathcal{T}} = \frac{\#\left\{(T_i, T_j) \,\middle|\, j \neq i, P_{\mathrm{KS}}(F_i, F_j) > (1-\alpha)\right\}}{\#\left\{(T_i, T_j) \,|\, j \neq i\right\}} = \frac{\#\text{matching pairs}}{\#\text{all pairs}}$$





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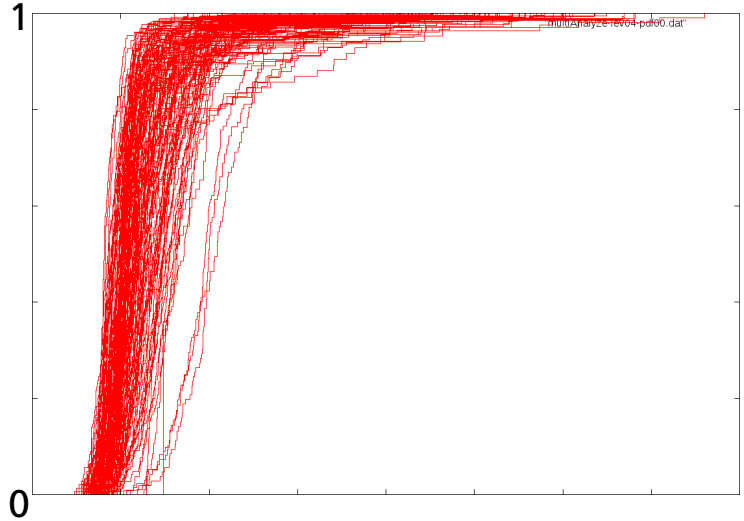
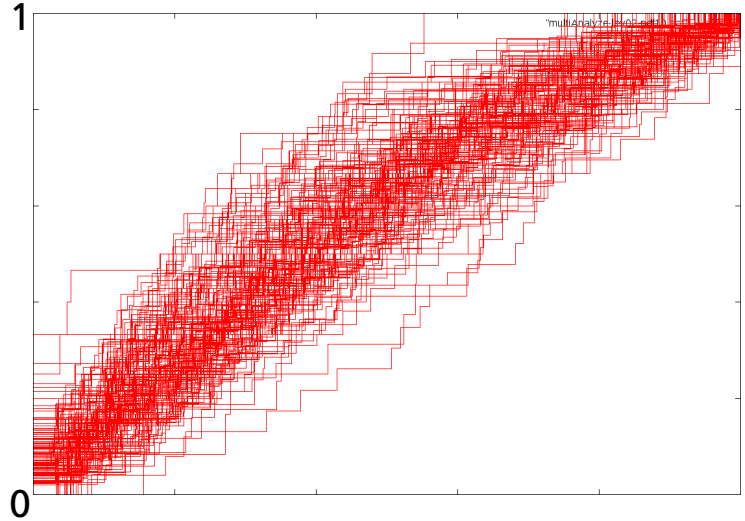
$m_{\mathcal{T}} \ll \alpha \approx 0 \Rightarrow$ non-invariant feature



$m_{\mathcal{T}} = 0.122$

$m_{\mathcal{T}} \approx \alpha \Rightarrow$ invariant feature



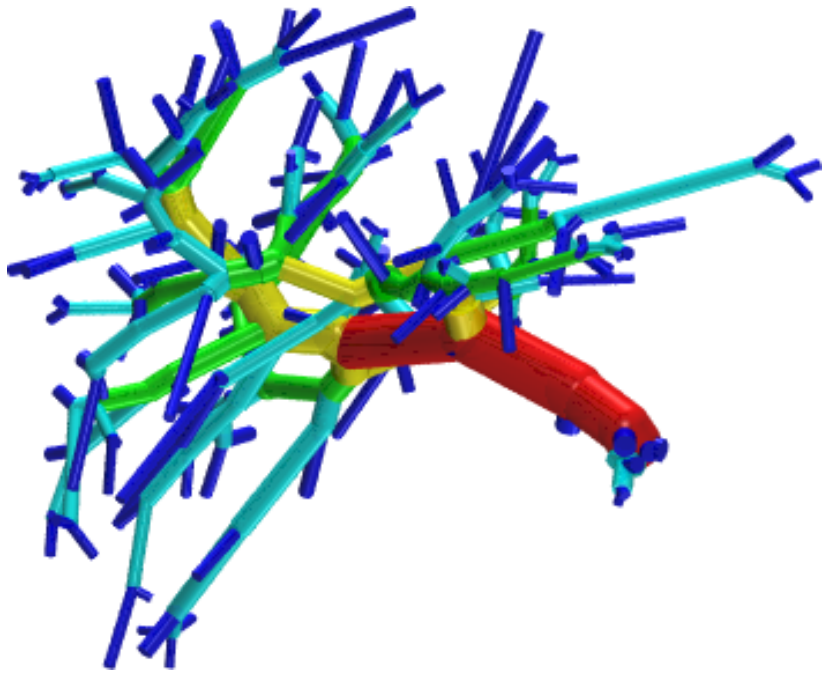
$m_{\mathcal{T}} = 0.914$

3. Statistical Comparison of Geometric Features

Averaging over Strahler* orders

$m_{\mathcal{T}}(s)$ per Strahler* order as before

$(\#\text{geom} = 1 \mid \#\text{Str*} = S \mid \#\text{Tr} = N)$



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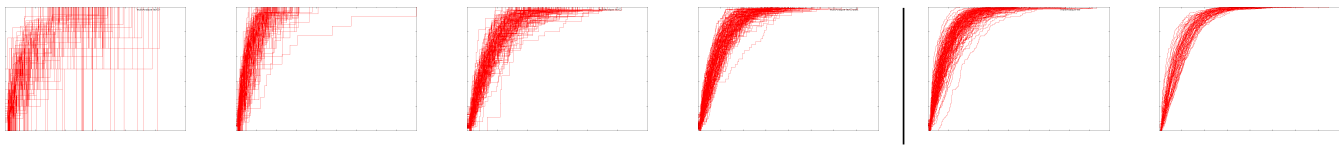
Strahler* order s	0	1	2	3	4	5
$m_{\mathcal{T}}(s)$	0.993	0.913	0.895	0.713	0.421	0.258



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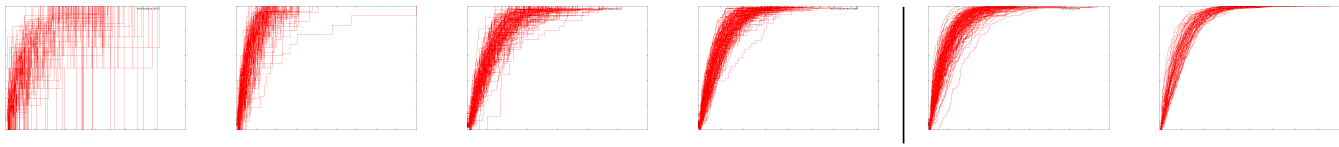
$m_{\mathcal{T}}(s)$	0.993	0.913	0.895	0.713	0.421	0.258
$n_e(s)$	5.38	16.81	31.67	67.26	134.04	277.32



$n_e(s)$ avg. # edges per tree considered

3. Statistical Comparison of Geometric Features

Averaging over Strahler* orders

$m_{\mathcal{T}}(s)$ per Strahler* order as before

$(\#\text{geom} = 1 \mid \#\text{Str*} = S \mid \#\text{Tr} = N)$

Strahler* order s	0	1	2	3	4	5
$m_{\mathcal{T}}(s)$	0.993	0.913	0.895	0.713	0.421	0.258
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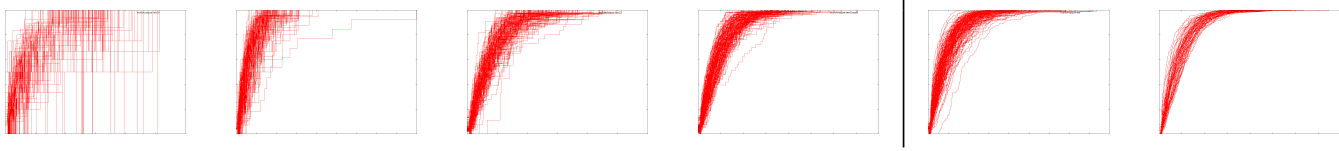

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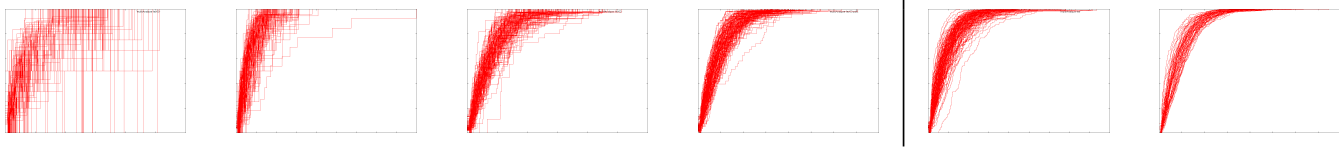

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Averaging

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(#geom = 1 | #Str* = S | #Tr = N)

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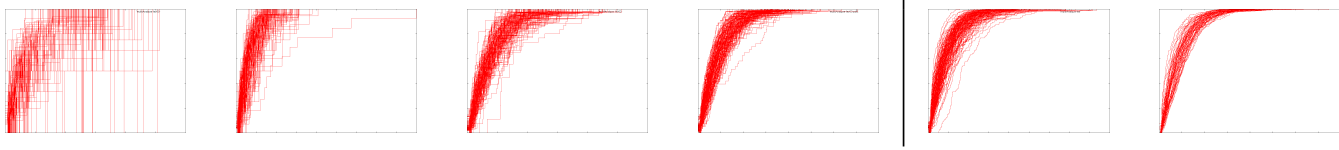

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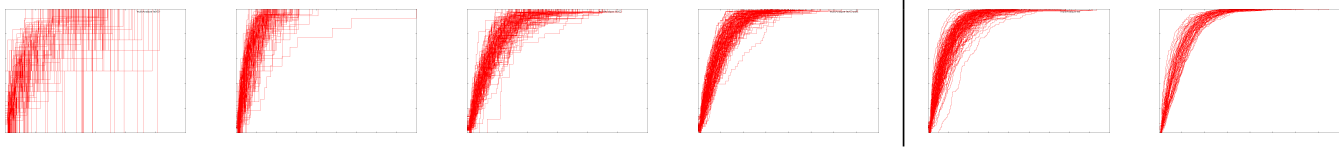

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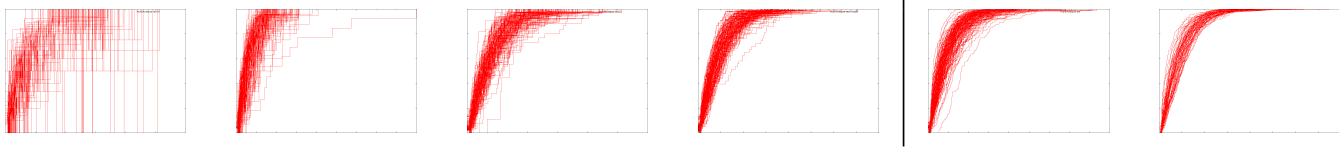


$n_e(s)$ avg. # edges per tree considered $n_t(s)$ # trees including order s

Averaging

$$\bar{m}_{\mathcal{T}} = \frac{\sum_{s \le s^*} w_s m_{\mathcal{T}}(s)}{\sum_{s \le s^*} w_s} = 0.801$$

with

$$w_s = n_e(s) \cdot n_t(s)$$

$$s^* = \max\left\{ s \,\middle|\, n_t(s+1) \ge \frac{1}{2} N \right\}$$

3. Statistical Comparison of Geometric Features

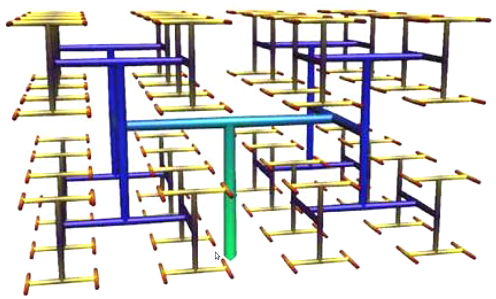
Results: Invariance of Geometric Features

Feature	PV match	HV match
radii r_0	0.291	0.281
radius decrease r_1/r_0	0.705	0.722
radius asymmetry r_2/r_1	0.789	0.731
bif. exponent γ	0.903	0.870
lengths l_0	0.801	0.813
length decrease l_1/l_0	0.969	0.950
length asymmetry l_2/l_1	0.930	0.946
angle φ_{01}	0.875	0.816
$\varphi_{01} + \varphi_{02}$	0.928	0.892
angle φ_{12}	0.933	0.888
parent inclination	0.940	0.878
daughter inclination	0.883	0.904

4. Algorithmic Generation of Vascular Structures
Contents

4. Algorithmic Generation of Vascular Structures
Overview of Generating Algorithms

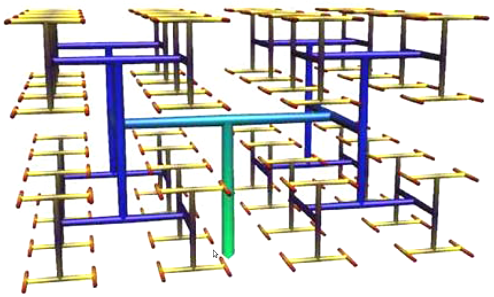
- artificial self-similar structures
  - easy to generate
  - very artificial visual appearance

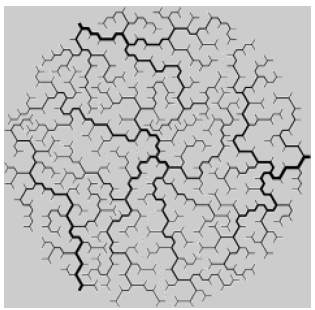


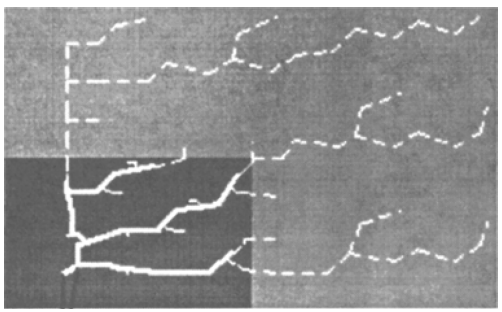
Hahn et al., Fractals 11 (2003)

4. Algorithmic Generation of Vascular Structures
Overview of Generating Algorithms

- artificial self-similar structures
  - easy to generate
  - very artificial visual appearance
- modeling angiogenesis
  - algorithmically complicated
  - grid-based algorithms lead to artificial visual appearance
  - tuning of geometric parameters?







Hahn et al., Fractals 11 (2003)
Gödde et al., Developmental Dynamics 220 (2001)
Nekka et al., Bulletin of Mathematical Biology 58 (1996)

4. Algorithmic Generation of Vascular Structures
Constrained Constructive Optimization

[Schreiner et al.]

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$$\pi \cdot \text{length} \cdot \text{radius}^{\lambda}$$

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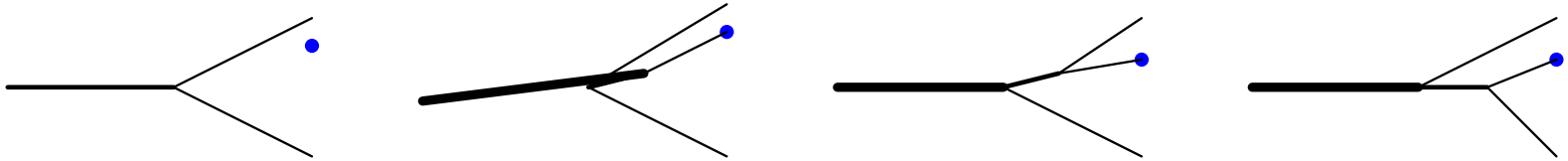

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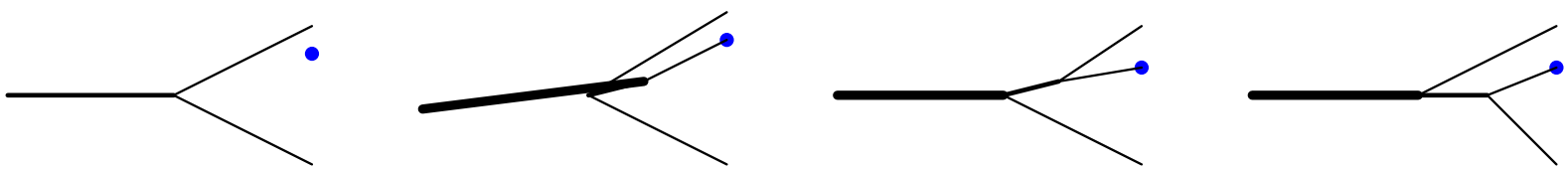

- non-convex organ shape treated via penalty term for bifurcation position

4. Algorithmic Generation of Vascular Structures
CCO Algorithmic Details

Edge radii determined by

- $r_0^\gamma = r_1^\gamma + r_2^\gamma$ with $\gamma = 3$ (Murray's law; later to be variable)
- r_1 and r_2 such that flow resistances of subtrees are balanced

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Serial and parallel connection of edges (Poiseuille's law for laminar flow)

$$R = \frac{8 \cdot \text{viscosity} \cdot \text{length}}{\pi \cdot \text{radius}^4}$$

with decreasing viscosity for radius < 150 µm (Fåhræus-Lindqvist effect)

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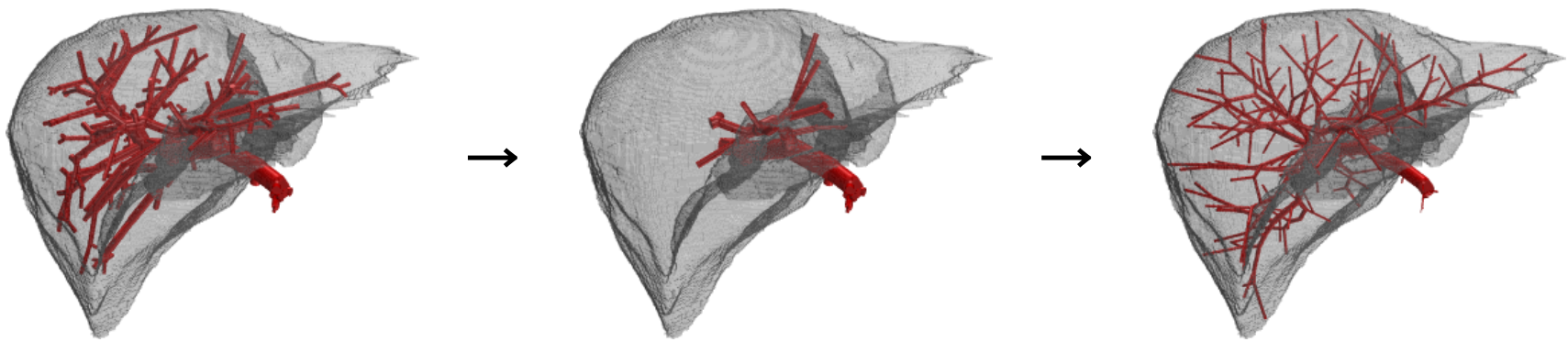
Computationally expensive optimization

- need to try different segments (constant-size neighborhood is enough)
- no analytic derivative wrt. position of bifurcation
- radii (not only of current bifurcation) depend on position

4. Algorithmic Generation of Vascular Structures

Validation: Prune & Regrow

- first validation step: check whether measurable complexity can be generated
- do not start from scratch but from pruned measured trees such as not to ignore anatomical coarse structures
- check whether parameters of the algorithm can be improved

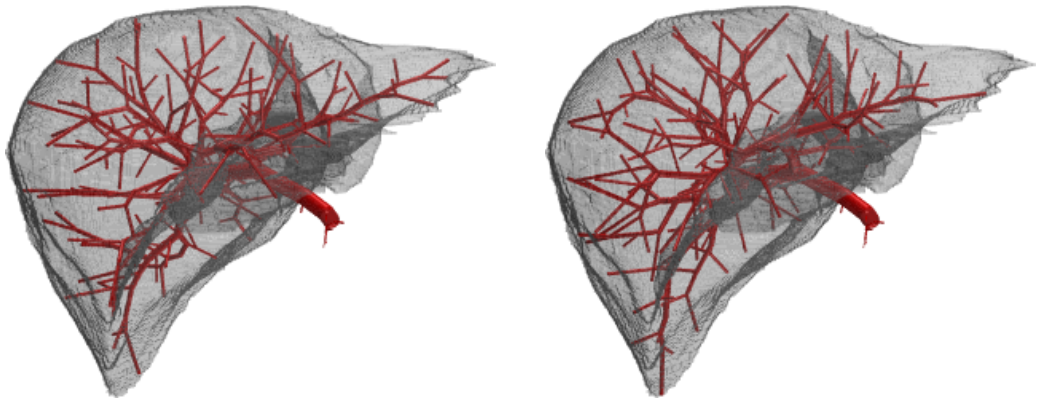


4. Algorithmic Generation of Vascular Structures

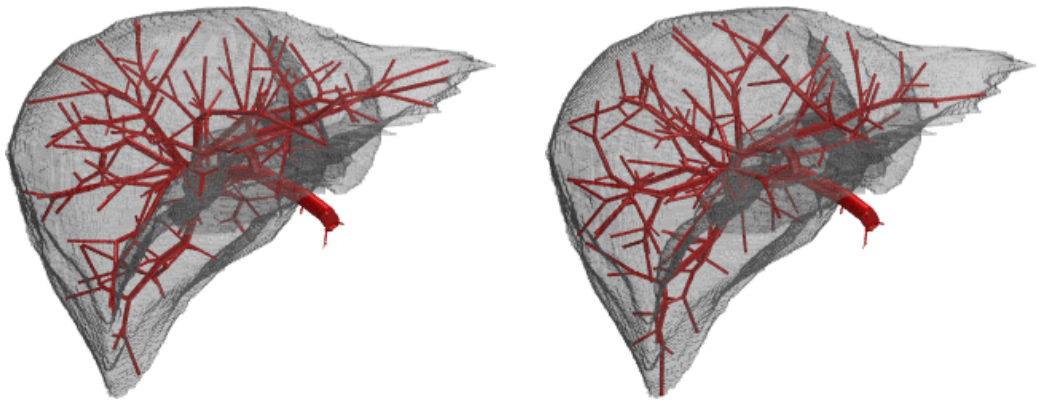
First Results and Parameter Improvements

Recall cost per edge $\pi \cdot \text{length} \cdot \text{radius}^{\lambda}$.

For $\lambda = 2.0$, we obtain



Are trees for $\lambda = 1.4$ better?



4. Algorithmic Generation of Vascular Structures

Comparison of Histograms: Cross-Match Ratios

$\mathcal{T} = \{T_0, \ldots, T_{N-1}\}$ measured (real) trees $(\#\text{geom} = 1 \mid \#\text{Str*} = 1 \mid \#\text{Tr} = M + N)$
$\mathcal{U} = \{U_0, \ldots, U_{M-1}\}$ artificial (generated) trees

$m_{\mathcal{U}} \backslash m_{\mathcal{T}}$	0	α
0	?	bad
α	artifact	check further

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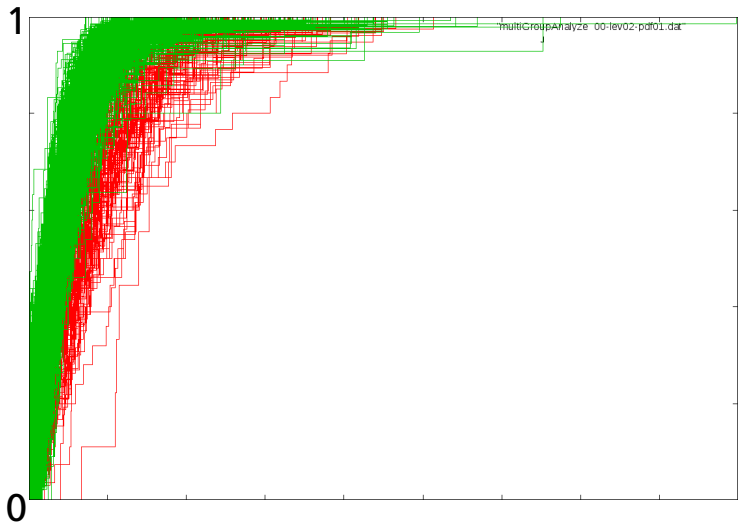
Let us consider measured vs. generated edge lengths …

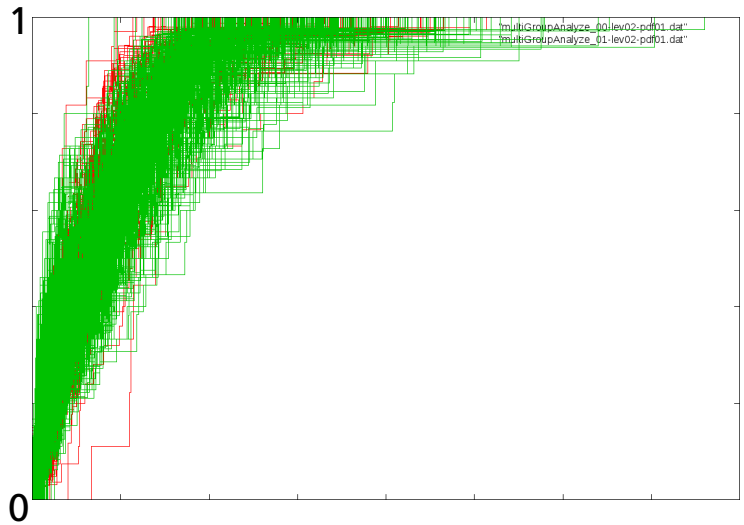
4. Algorithmic Generation of Vascular Structures

Comparison of Histograms: Cross-Match Ratios

Define 'cross match ratio' (similar to before) $(\#\text{geom} = 1 \mid \#\text{Str}^* = 1 \mid \#\text{Tr} = M + N)$

$$m_{\mathcal{TU}}(s) = \frac{\#\left\{(T_i, U_j) \,\middle|\, P_{\text{KS}}(F_i, F_j) > (1-\alpha)\right\}}{\#\{(T_i, U_j)\}} = \frac{\#\text{matching pairs}}{\#\text{all pairs}}$$





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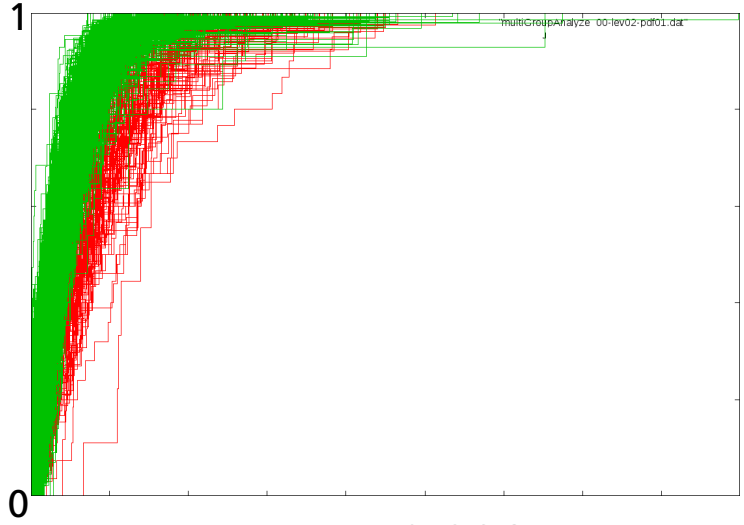
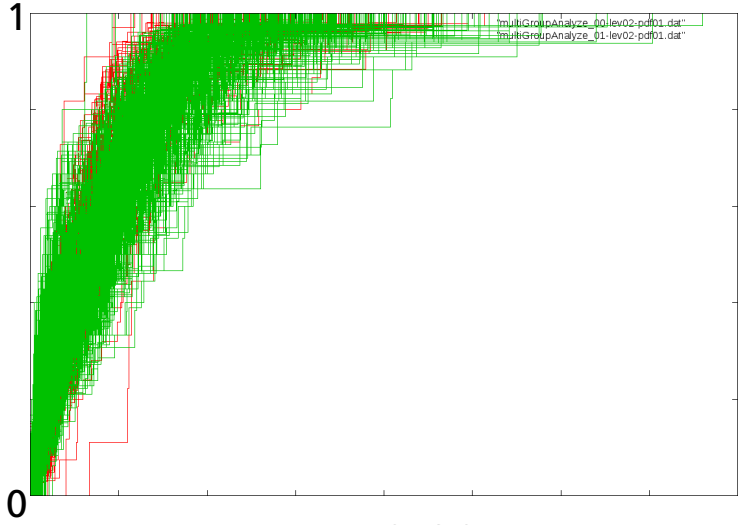
$m_{\mathcal{TU}} \ll \alpha \Rightarrow$ non-reproduced invariant



$m_{\mathcal{TU}} = 0.396$

$m_{\mathcal{T}} \approx \alpha \Rightarrow$ reproduced invariant



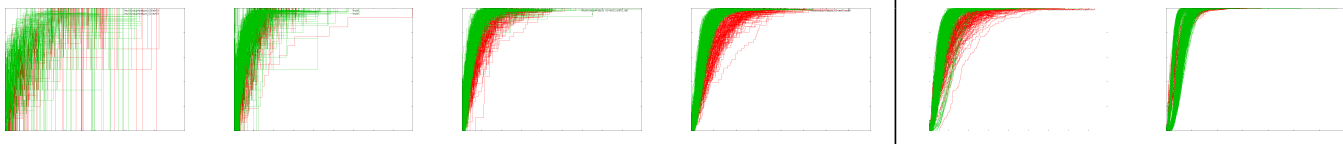
$m_{\mathcal{T}} = 0.904$

4. Algorithmic Generation of Vascular Structures

Averaging over Strahler* orders

$(\#\text{geom} = 1 \mid \#\text{Str*} = S \mid \#\text{Tr} = M + N)$

Strahler* order s	0	1	2	3	4	5
$m_{\mathcal{TU}}(s)$	0.969	0.694	0.396	0.162	0.045	0.004
$n_e, m_e(s)$	5.38, 7.97	16.81, 21.36	31.67, 57.39	67.26, 120.10	134.04, 222.42	277.32, n/a
$n_t, m_t(s)$	167, 710	167, 710	167, 710	165, 710	150, 710	62, 0



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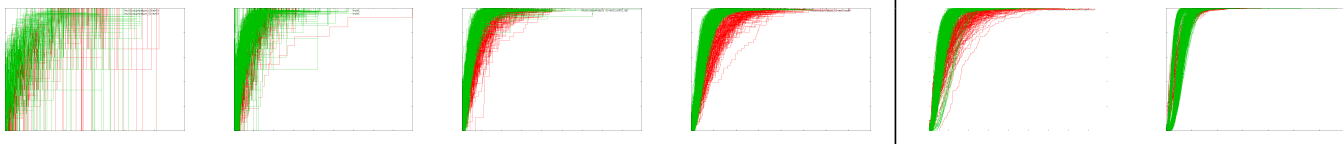

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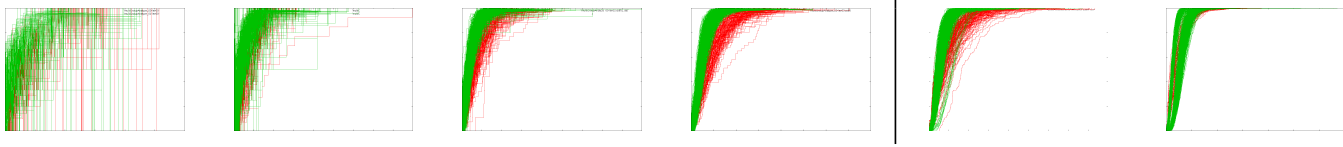

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$$\bar{m}_{\mathcal{TU}} = \frac{\sum w_s m_{\mathcal{TU}}(s)}{\sum w_s}$$

with

$$w_s = \sqrt{(n_e(s) n_t(s)) \cdot (m_e(s) m_t(s))}$$

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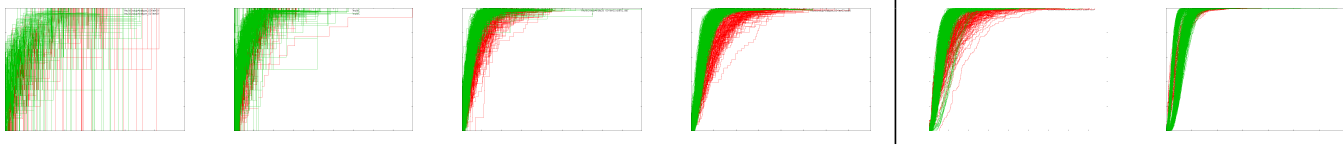
$n_t, m_t(s)$	167, 710	167, 710	167, 710	165, 710	150, 710	62, 0



Recall n_e = avg. # edges per tree considered, n_t = # trees including order s

Averaging

$$\bar{m}_{\mathcal{TU}} = \frac{\sum_{s \le s^*} w_s m_{\mathcal{TU}}(s)}{\sum_{s \le s^*} w_s}$$

with

$$w_s = \sqrt{(n_e(s) n_t(s)) \cdot (m_e(s) m_t(s))}$$

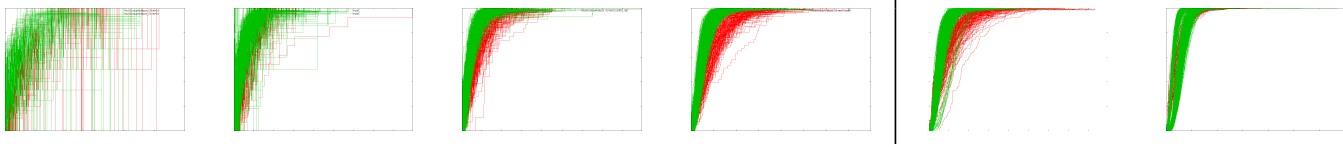
$$s^* = \max\left\{ s \,\middle|\, n_t(s+1) \ge \frac{1}{2} N \wedge m_t(s+1) \ge \frac{1}{2} M \right\}$$

4. Algorithmic Generation of Vascular Structures

Averaging over Strahler* orders

$(\#\text{geom} = 1 \mid \#\text{Str*} = S \mid \#\text{Tr} = M + N)$

Strahler* order s	0	1	2	3	4	5
$m_{\mathcal{TU}}(s)$	0.969	0.694	0.396	0.162	0.045	0.004
$n_e, m_e(s)$	5.38, 7.97	16.81, 21.36	31.67, 57.39	67.26, 120.10	134.04, 222.42	277.32, n/a
$n_t, m_t(s)$	167, 710	167, 710	167, 710	165, 710	150, 710	62, 0



Recall n_e = avg. # edges per tree considered, n_t = # trees including order s

Averaging

$$\bar{m}_{\mathcal{TU}} = \frac{\sum_{s \le s^*} w_s m_{\mathcal{TU}}(s)}{\sum_{s \le s^*} w_s} = 0.323$$

with

$$w_s = \sqrt{(n_e(s) n_t(s)) \cdot (m_e(s) m_t(s))}$$

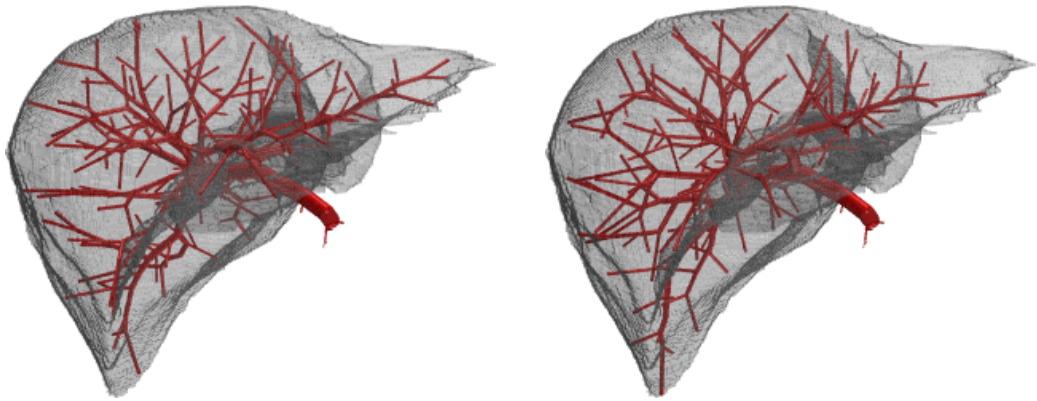
$$s^* = \max \left\{ s \,\middle|\, n_t(s+1) \ge \frac{1}{2} N \wedge m_t(s+1) \ge \frac{1}{2} M \right\}$$

4. Algorithmic Generation of Vascular Structures

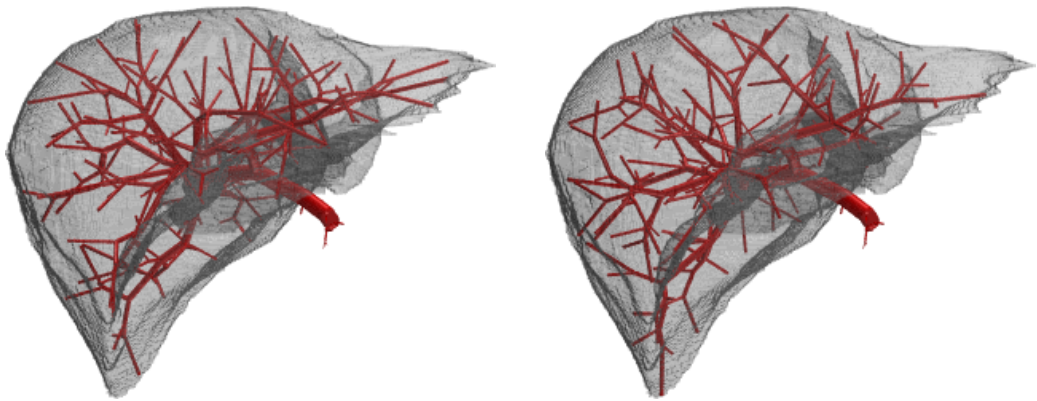
First Results and Parameter Improvements

Recall cost per edge $\pi \cdot \text{length} \cdot \text{radius}^{\lambda}$.

For $\lambda = 2.0$, we obtain



Are trees for $\lambda = 1.4$ better?



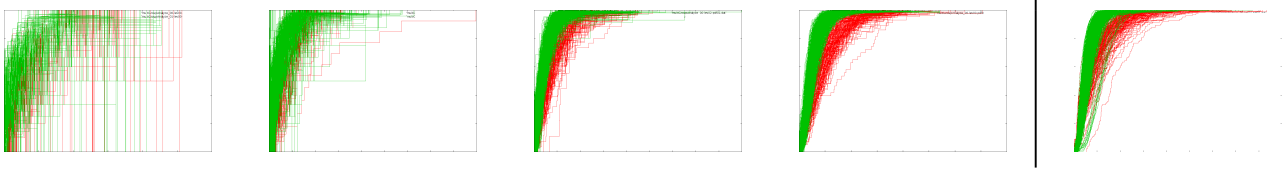
4. Algorithmic Generation of Vascular Structures

First Results: Resulting Edge Lengths

$(\#\text{geom} = 1 \mid \#\text{Str*} = S \mid \#\text{Tr} = M + N)$

For $\lambda = 2.0$, we obtain (red: real, green: generated trees) $\bar{m}_{\mathcal{TU}} = \mathbf{0.323}$

Strahler* order s	0	1	2	$s^* = 3$	4
$m_{\mathcal{TU}}(s)$	0.969	0.694	0.396	0.162	0.045
$n_e, m_e(s)$	5.38, 7.97	16.81, 21.36	31.67, 57.39	67.26, 120.10	134.04, 222.42
$n_t, m_t(s)$	167, 710	167, 710	167, 710	165, 710	150, 710



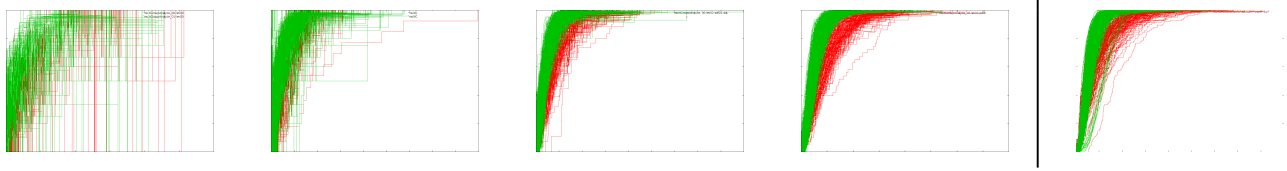
4. Algorithmic Generation of Vascular Structures

First Results: Resulting Edge Lengths

$(\#\text{geom} = 1 \mid \#\text{Str}^* = S \mid \#\text{Tr} = M + N)$

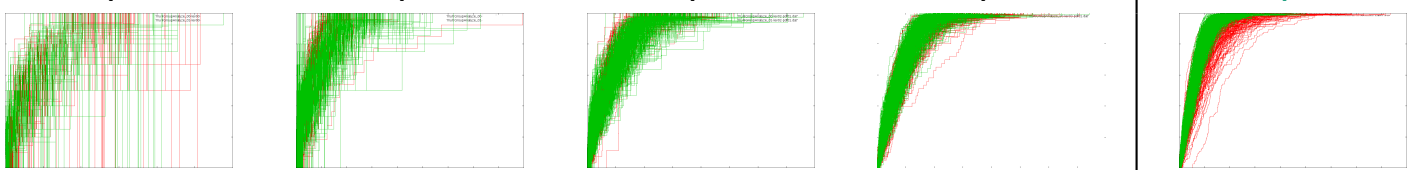
For $\lambda = 2.0$, we obtain (red: real, green: generated trees) $\bar{m}_{\mathcal{TU}} = 0.323$

Strahler* order s	0	1	2	$s^* = 3$	4
$m_{\mathcal{TU}}(s)$	0.969	0.694	0.396	0.162	0.045
$n_e, m_e(s)$	5.38, 7.97	16.81, 21.36	31.67, 57.39	67.26, 120.10	134.04, 222.42
$n_t, m_t(s)$	167, 710	167, 710	167, 710	165, 710	150, 710



Matches for $\lambda = 1.4$ are better: $\bar{m}_{\mathcal{TU}} = 0.842$

$m_{\mathcal{TU}}(s)$	0.992	0.935	0.904	0.780	0.309
$n_e, m_e(s)$	5.38, 5.38	16.81, 10.21	31.67, 23.71	67.26, 56.27	134.04, 133.02
$n_t, m_t(s)$	167, 710	167, 710	167, 710	165, 710	150, 710

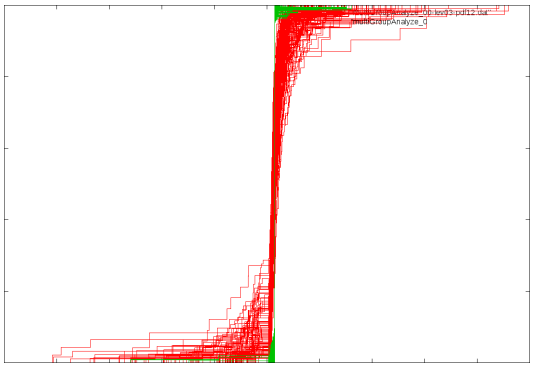


5. Outlook
Contents

5. Outlook

Systematic Parameter Improvements

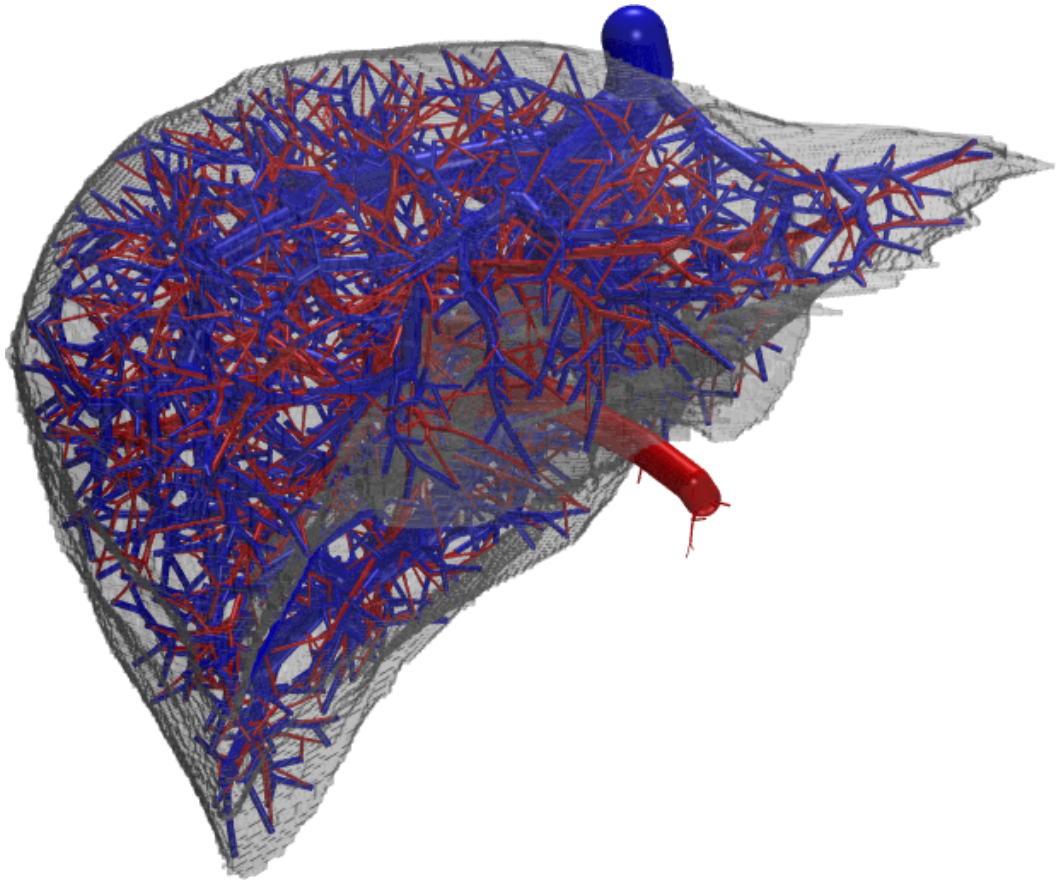
- cost exponent λ in edge cost $\pi \cdot \text{length} \cdot \text{radius}^{\lambda}$.
- 'bifurcation exponent' $r_0^{\gamma} = r_1^{\gamma} + r_2^{\gamma}$ should follow natural distribution $\not\equiv 3$



- joint optimization of parameters to make 'all' geometric properties realistic

5. Outlook

Multiple Vascular Trees



- implement non-penetration for pair of trees

5. Outlook

Next Steps

- building block in multiscale simulation
  not only classical Coinaud Segments

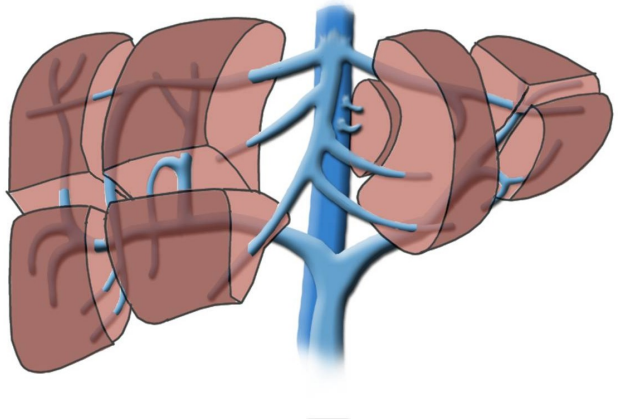


image from healcentral.org © Royal College of Surgeons of Ireland

- connections between
  - organism/whole body scale
  - lobular scale

  as part of the Virtual Liver Network
- other species
- could also treat other organs/structures
- simulate e.g. flow through vascular structures

5. Outlook

Acknowledgments

Funded by:

EUROPEAN UNION:
Investing in your future
European Regional Development Fund

5. Outlook

References

- Fåhræus et al., *The Viscosity of the Blood in Narrow Capillary Tubes*, American Journal of Physiology **96** (1931)
- Strahler, *Quantitative analysis of watershed geomorphology*, Transactions of the American Geophysical Union **8** (1957)
- Schreiner et al., *Computer-Optimization of Vascular Trees*, IEEE Transactions on Biomedical Engineering **40** (1993)
- Neumann et al., *Computer simulation of coronary arterial trees*, Advances in Engineering Software **28** (1997)
- Karch et al., *A three-dimensional model for arterial tree representation, generated by constrained constructive optimization*, Computers in Biology and Medicine **29** (1999)
- Selle et al., *Analysis of vasculature for liver surgical planning*, IEEE Transactions on Medical Imaging **21** (2002)
- Op Den Buijs et al., *Branching Morphology of the Rat Hepatic Portal Vein Tree: A Micro-CT Study*, Annals of Biomedical Engineering **34** (2006)
- Schreiner et al., *Optimized arterial trees supplying hollow organs*, Medical Engineering & Physics **28** (2006)