Numerical Simulation of Transport and Diffusion in Drainage Media

Ole Schwen
Institute for Numerical Simulation
University of Bonn

Bonn, 2006-01-26

Outline

- introduction, medical problem
- generate appropriate geometry
- advection through blood vessels using ELLAM
- diffusion in tissue
- coupling

Introduction

- RF ablation of tumors:
- numerical simulation of thermal processes
- heat diffusion in tissue
- transport of heat by blood flow through vessel system
- develop tools in 2D to keep things simpler

Introduction

- RF ablation of tumors:
- numerical simulation of thermal processes
- heat diffusion in tissue
- transport of heat by blood flow through vessel system
- develop tools in 2D to keep things simpler

Outline

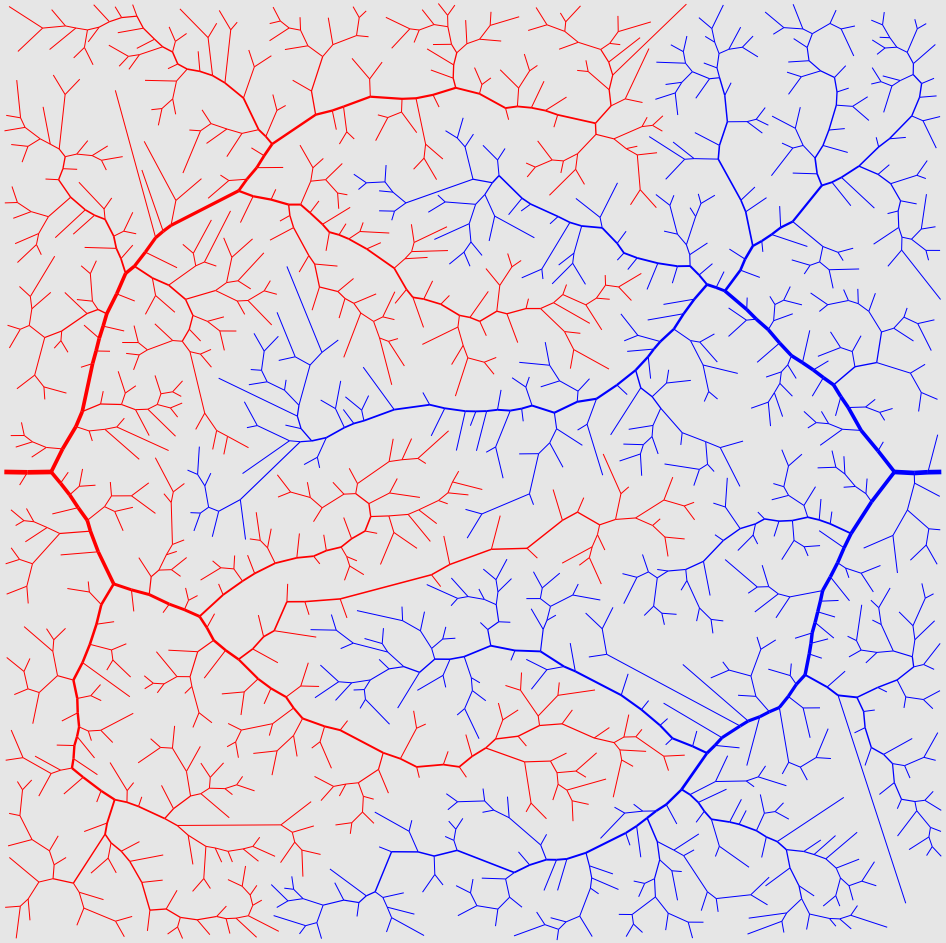
2D Vessel Trees
- Geometric Structure
- Flow Velocities

Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

Pairs of Vessel Trees (geometric structure)

geometric structure can be constructed using CCO (Schreiner et al. 1997).



But what about flow velocities? ▷

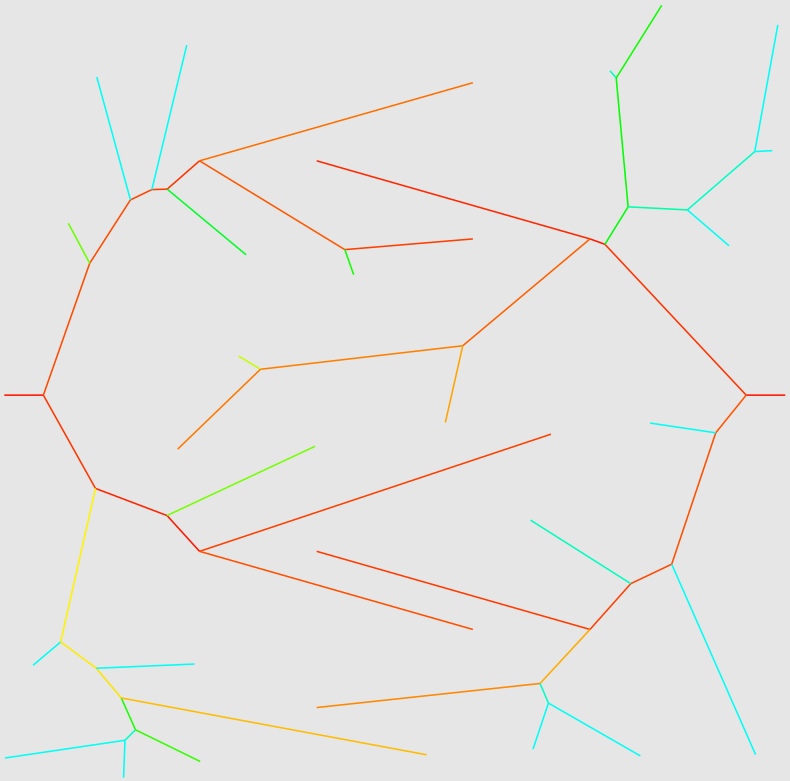
Outline

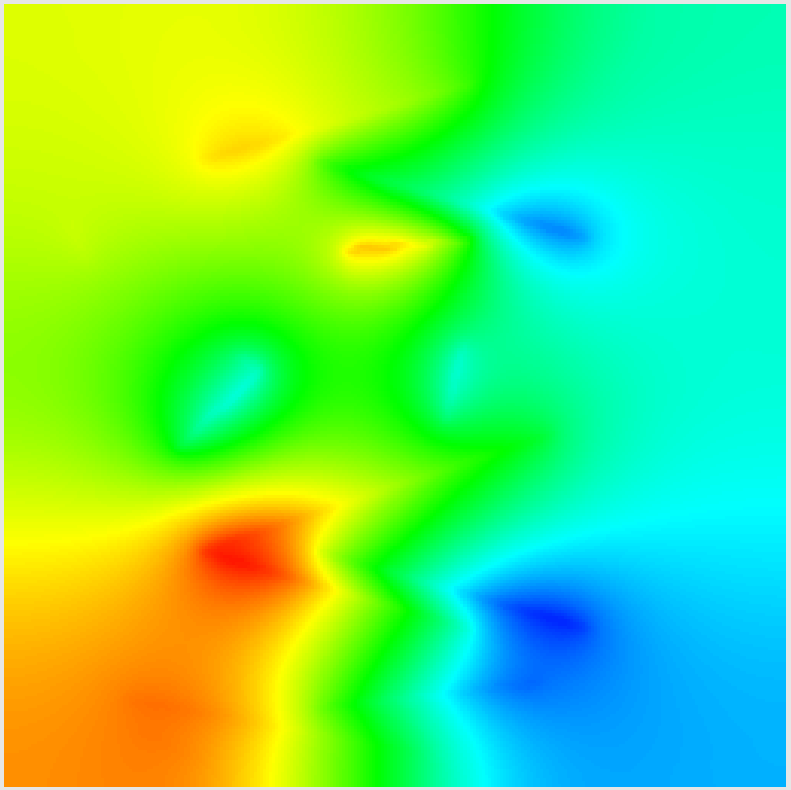
2D Vessel Trees
- Geometric Structure
- Flow Velocities

Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

Flow Velocities and Pressure Profile: Results





Advection in Vessel Trees: Overview

- single segments
- $\prec$ and $\succ$ bifurcations
- terminal segments
- trees

$\triangleright$

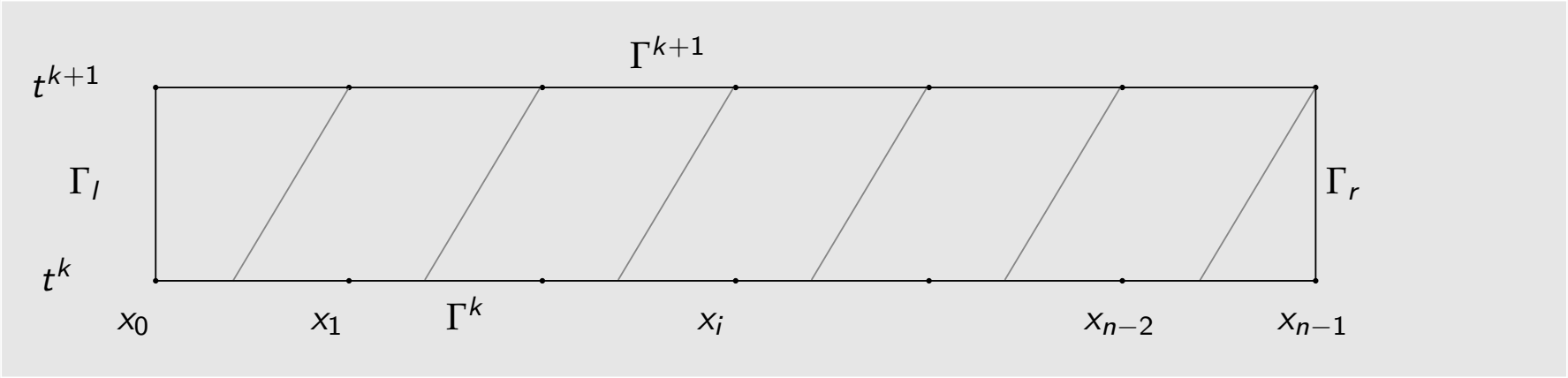
Outline

2D Vessel Trees
- Geometric Structure
- Flow Velocities

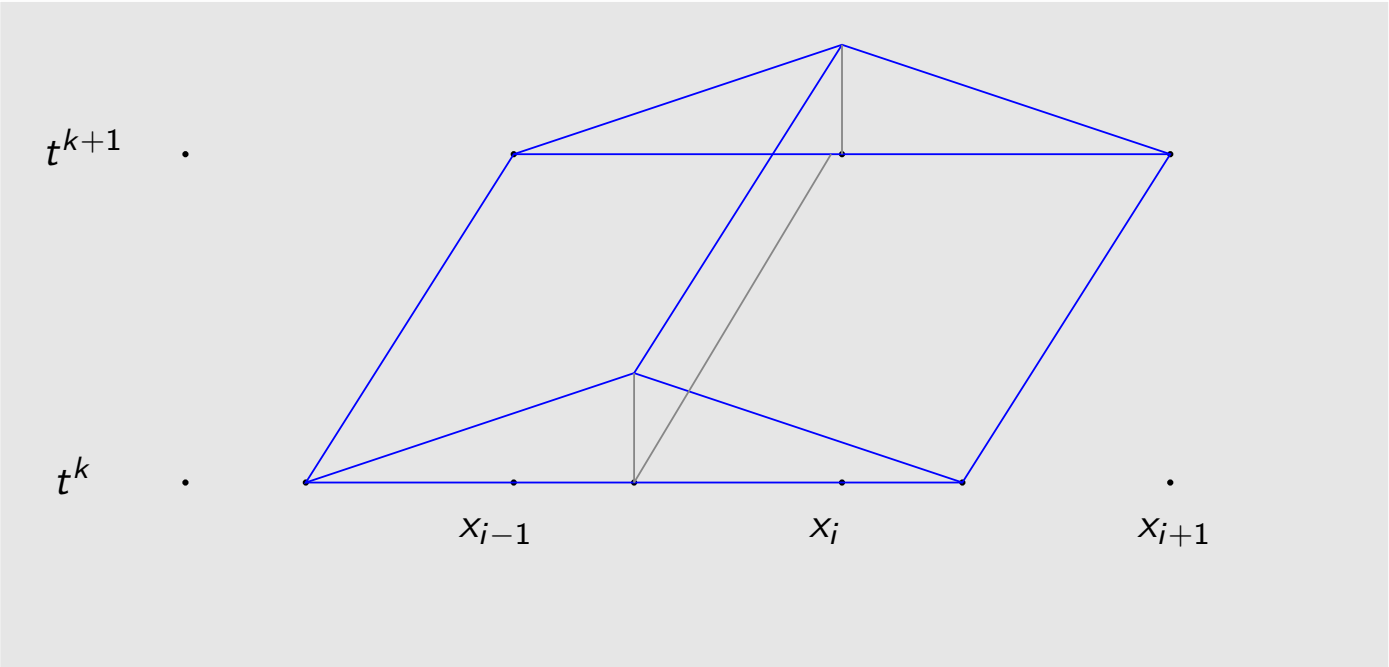
Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

Domains for ELLAM timestep



Test Functions



restriction: $v < 1$ $\triangleright$

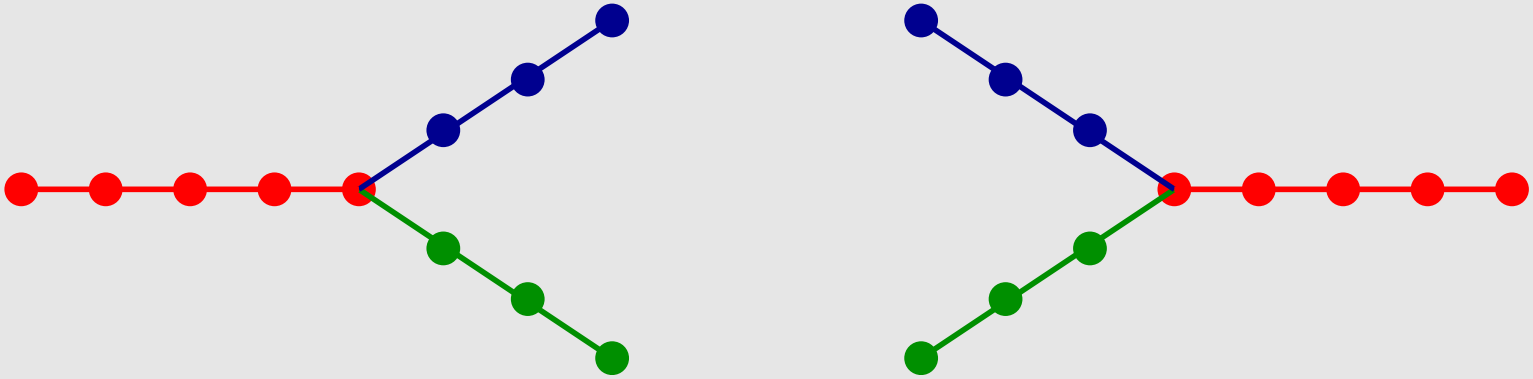
Outline

2D Vessel Trees
- Geometric Structure
- Flow Velocities

Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

Location of Unknowns



▷

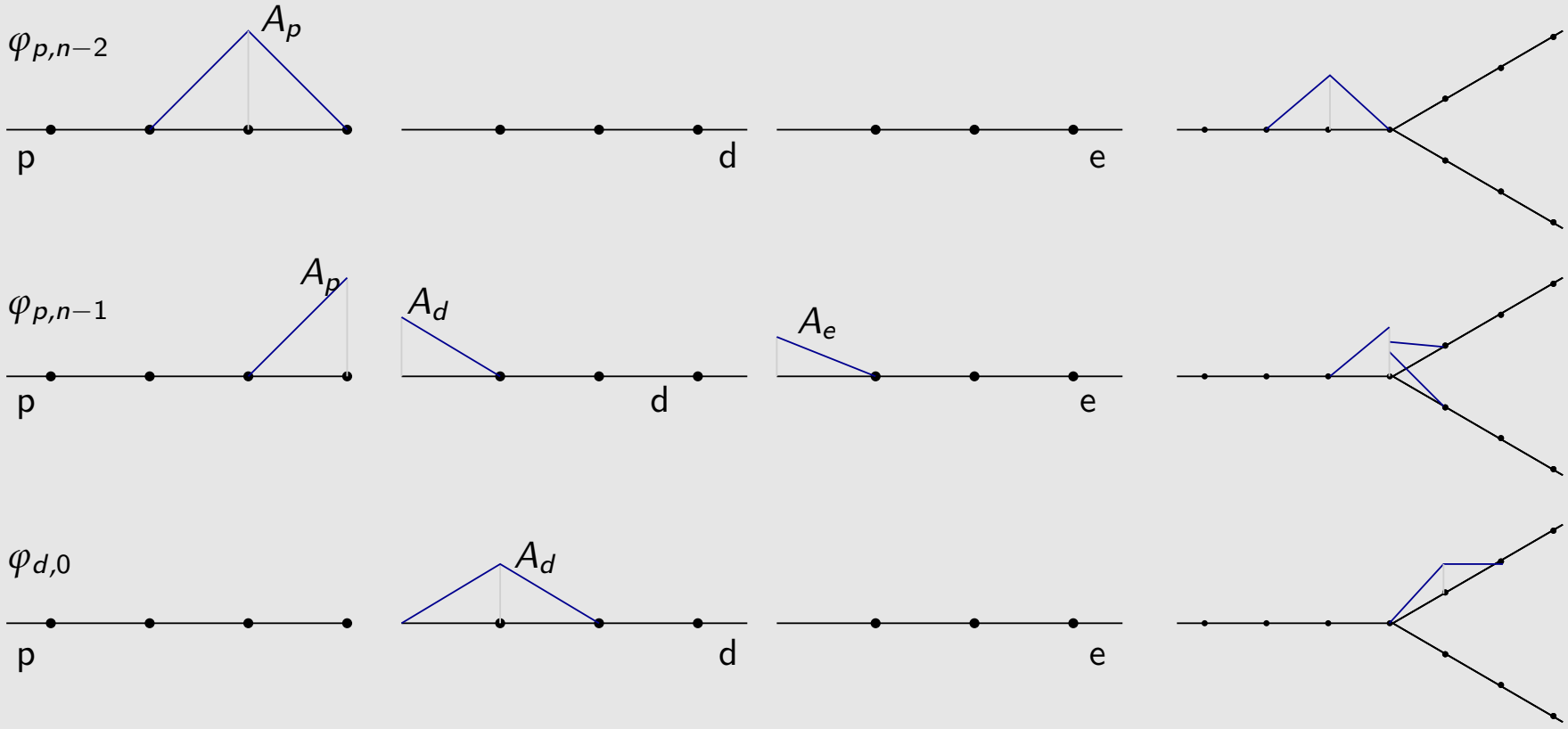
Outline

2D Vessel Trees
- Geometric Structure
- Flow Velocities

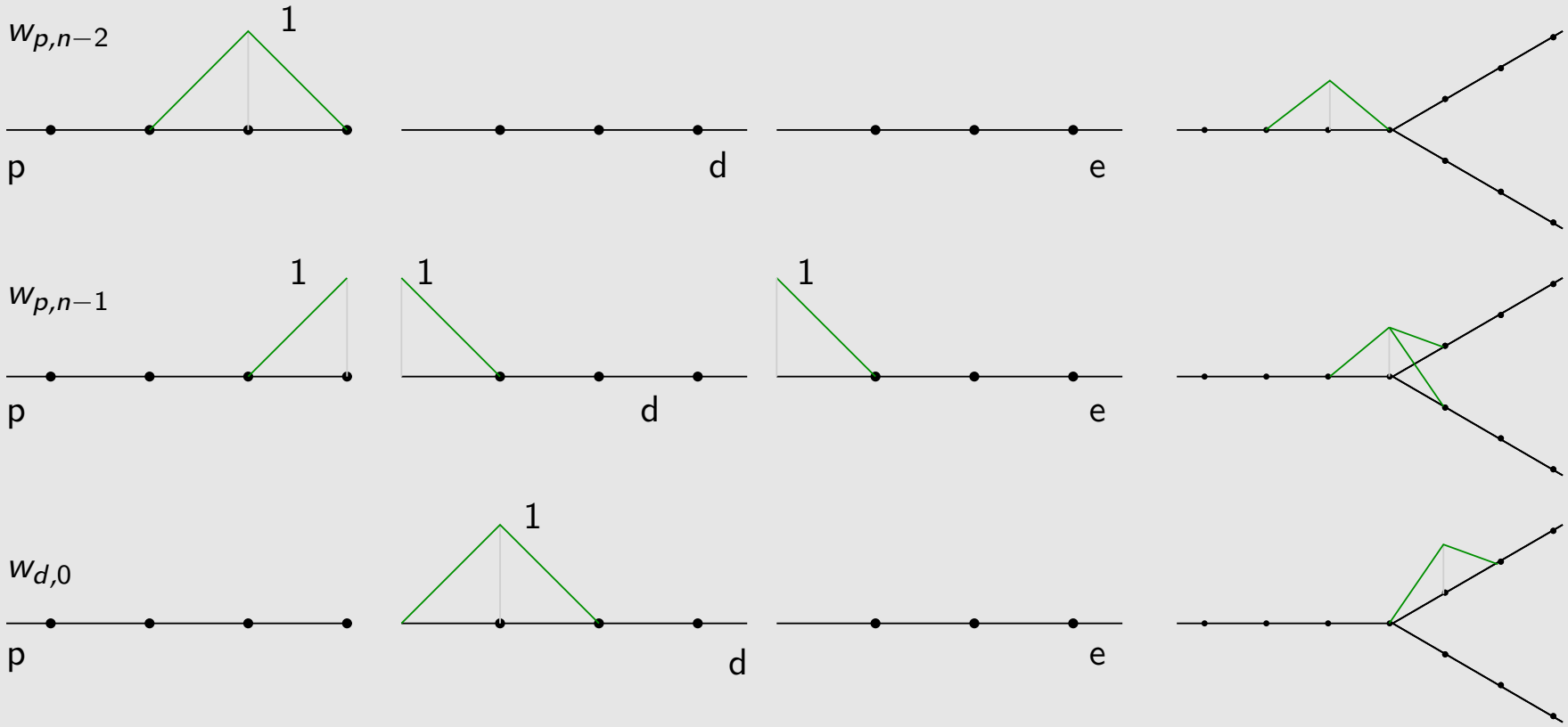
Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

Basis Functions at Arterial Bifurcations

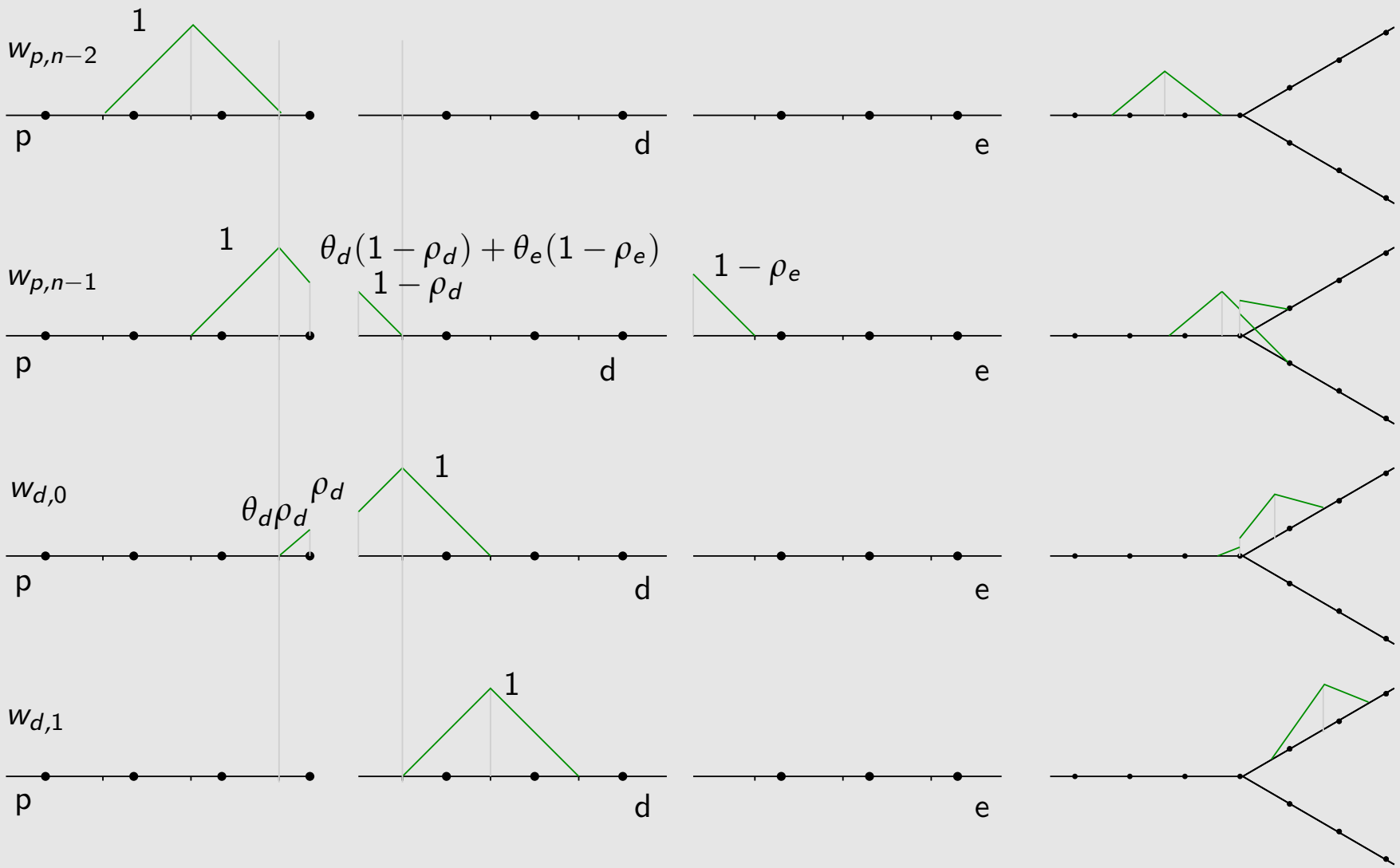


Test Functions at Arterial Bifurcations



$$\theta_d = \frac{A_d \cdot v_d}{A_p \cdot v_p}, \theta_e = \frac{A_e \cdot v_e}{A_p \cdot v_p}, \theta_d + \theta_e = 1$$

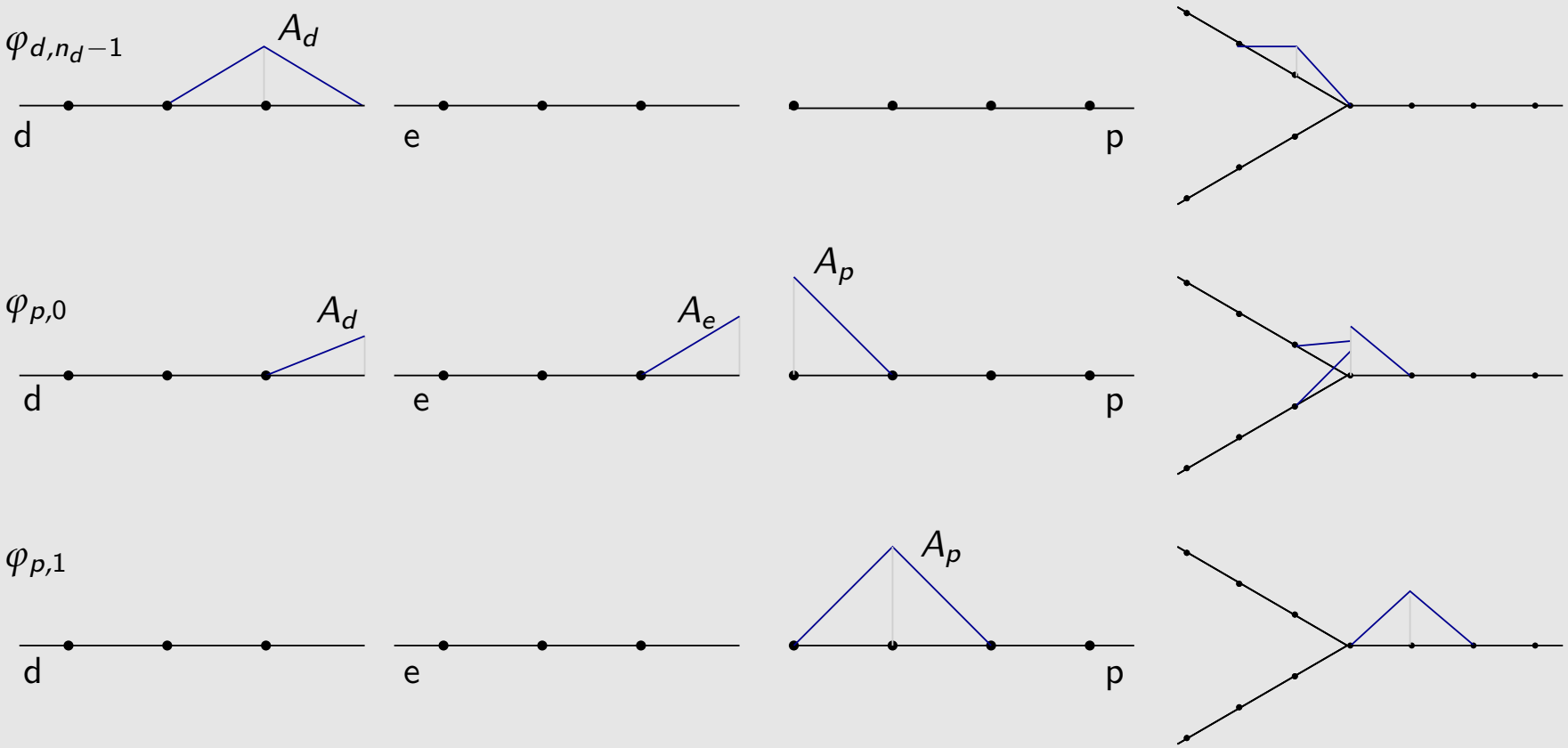
Test Functions at Arterial Bifurcations



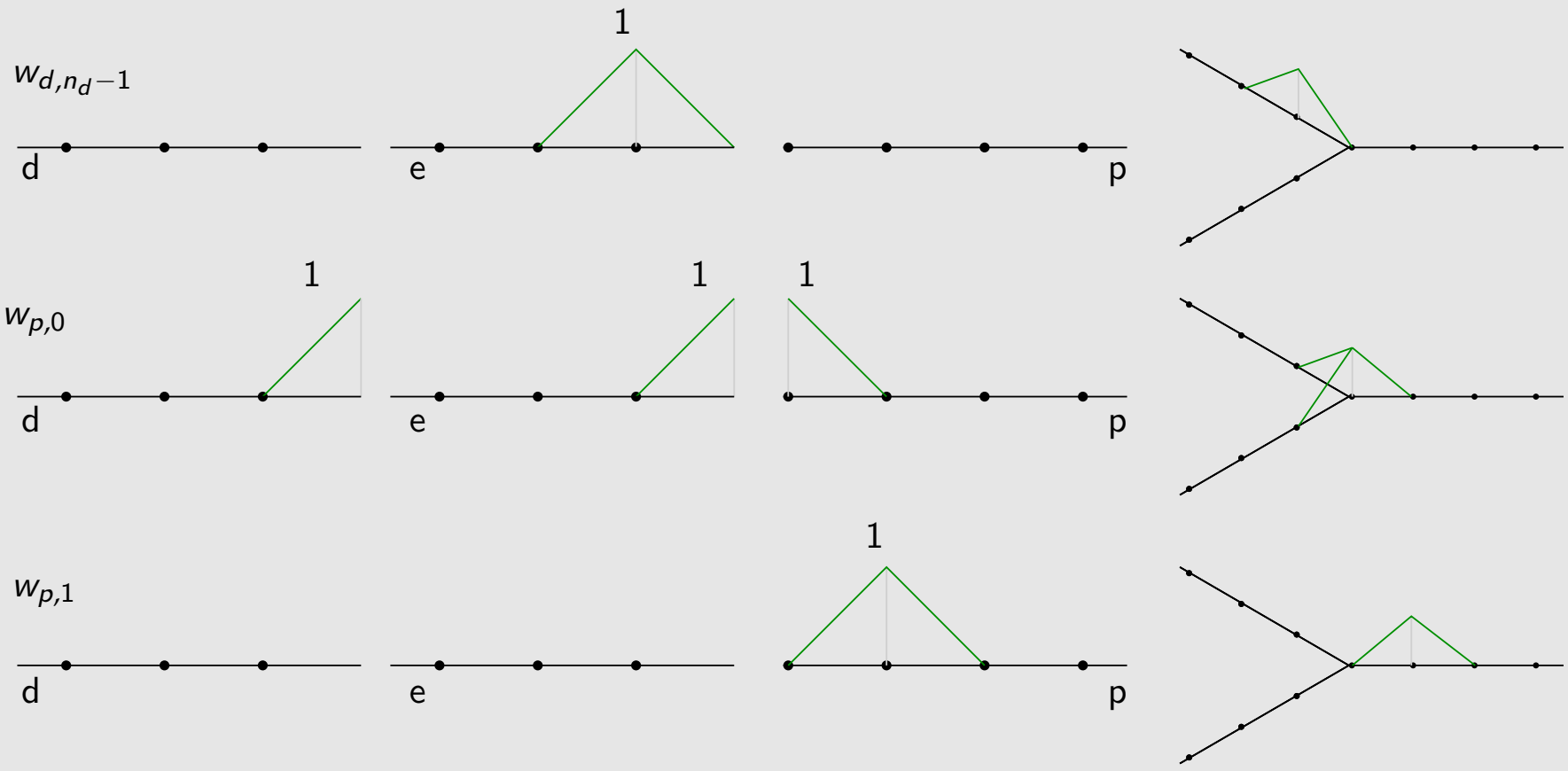
$$\theta_d = \frac{A_d \cdot v_d}{A_p \cdot v_p}, \theta_e = \frac{A_e \cdot v_e}{A_p \cdot v_p}, \theta_d + \theta_e = 1$$

universitätbonn

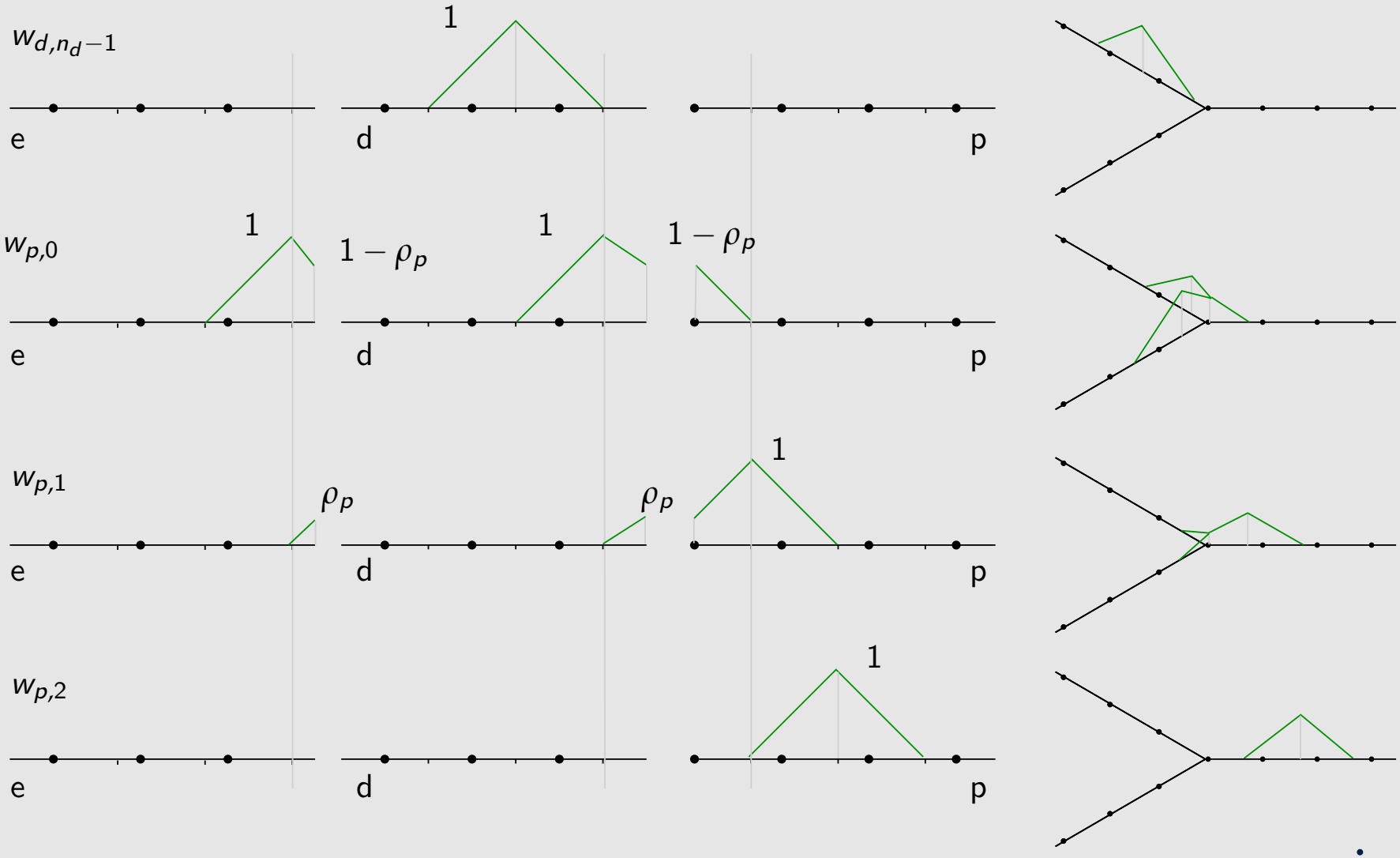
Basis Functions at Venous Bifurcations



Test Functions at Venous Bifurcations



Test Functions at Venous Bifurcations



Advection at $\prec$ bifurcation

movie: splitting of a temperature profile

Advection at $\succ$ bifurcation

movie: continuous combination of temperature profiles

Advection at $\succ$ bifurcation

movie: discontinuous combination of temperature profiles

Outline

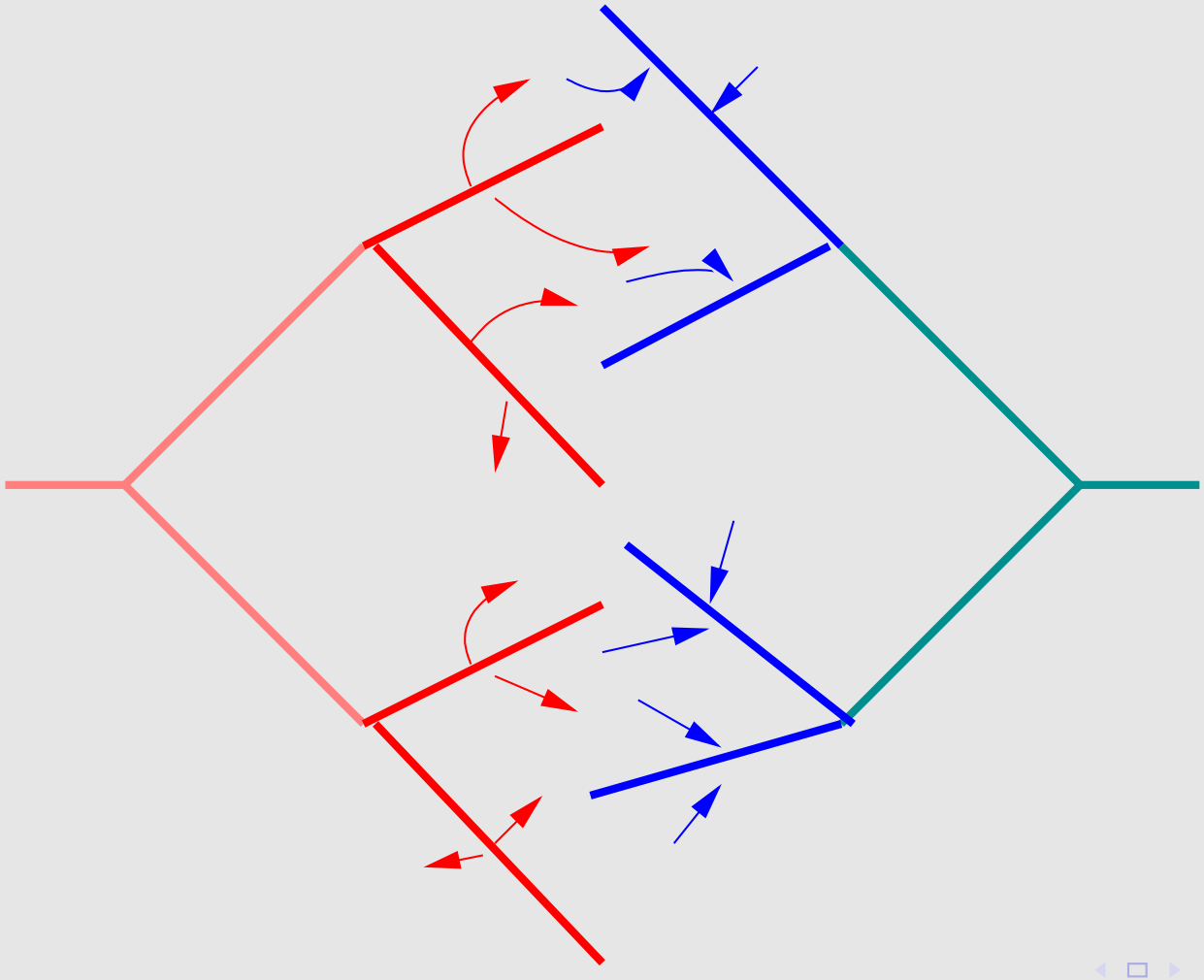
2D Vessel Trees
- Geometric Structure
- Flow Velocities

Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

Terminal Segments

- Outflow of mass out of arterial terminal segments (and only these)
- Inflow of mass into venous terminal segments (and only these)



RHS source terms

Arterial terminal segments:

$$\partial_t u(t,x) + v \cdot \partial_x u(t,x) = - v \cdot \frac{u(t,x)}{\ell - x} \tag{1}$$

Venous terminal segments:

$$\partial_t u(t,x) + v \cdot \partial_x u(t,x) = \frac{A_\emptyset v}{\ell} \cdot u_{\text{tissue}}(t,x) \tag{2}$$

RHS source terms

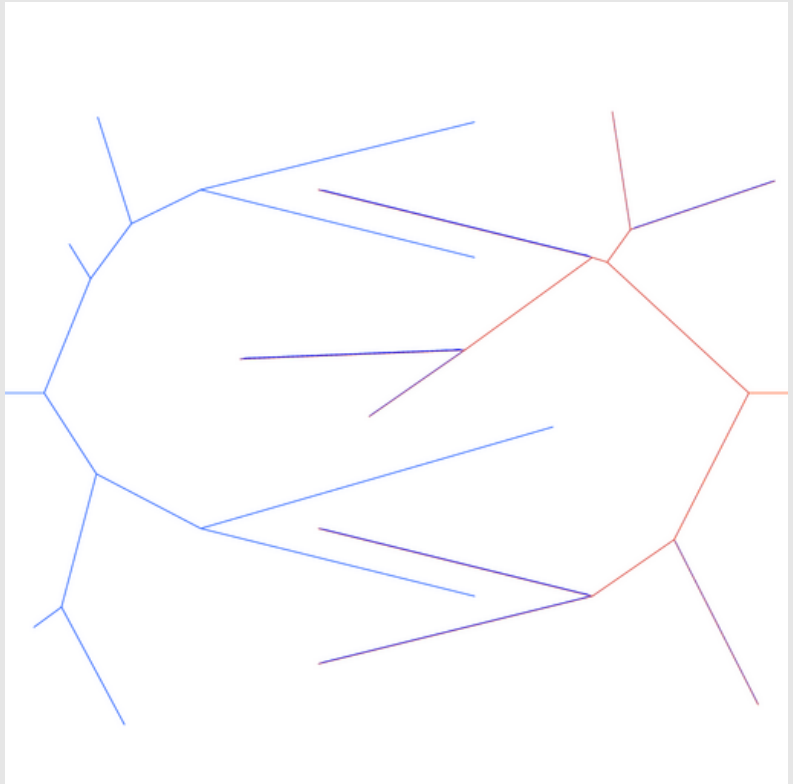
Arterial terminal segments:

$$\partial_t u(t,x) + v \cdot \partial_x u(t,x) = - v \cdot \frac{u(t,x)}{\ell - x} \tag{1}$$

Venous terminal segments:

$$\partial_t u(t,x) + v \cdot \partial_x u(t,x) = \frac{A_\varnothing v}{\ell} \cdot u_{\text{tissue}}(t,x) \tag{2}$$

Advection: Temperature Content Profiles on a Tree



Outline

2D Vessel Trees
- Geometric Structure
- Flow Velocities

Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

universitätbonn

Heat conduction in Tissue

Heat conduction ($u(t,x)$ temperature, κ conductivity):

$$\partial_t u(t,x) - \kappa \Delta_x u(t,x) = f(t,x)$$

initial values, zero Neumann boundary condition.
f on vessel trees (1D line segments), PDE meant in distributional sense. $\triangleright$

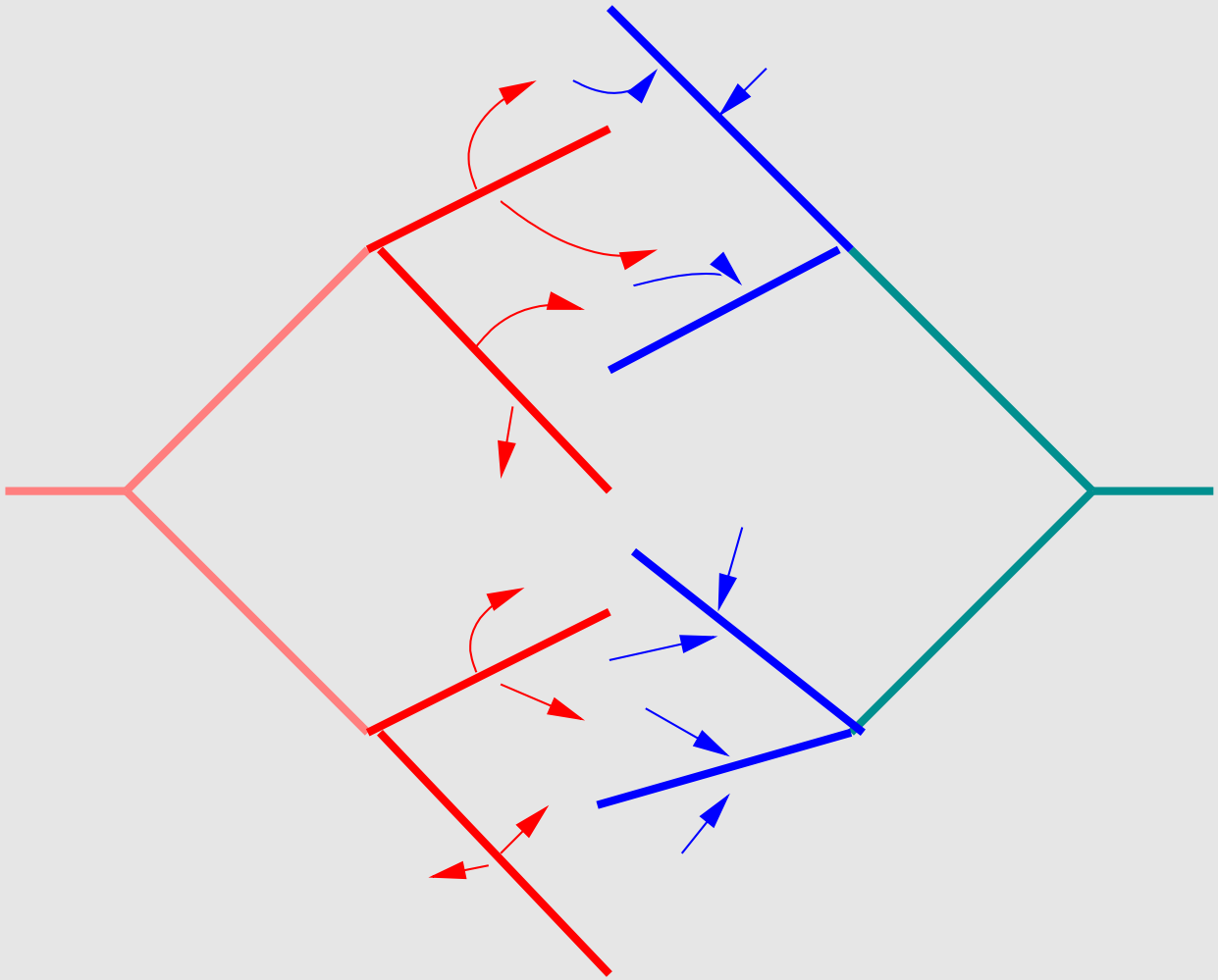
Outline

2D Vessel Trees
- Geometric Structure
- Flow Velocities

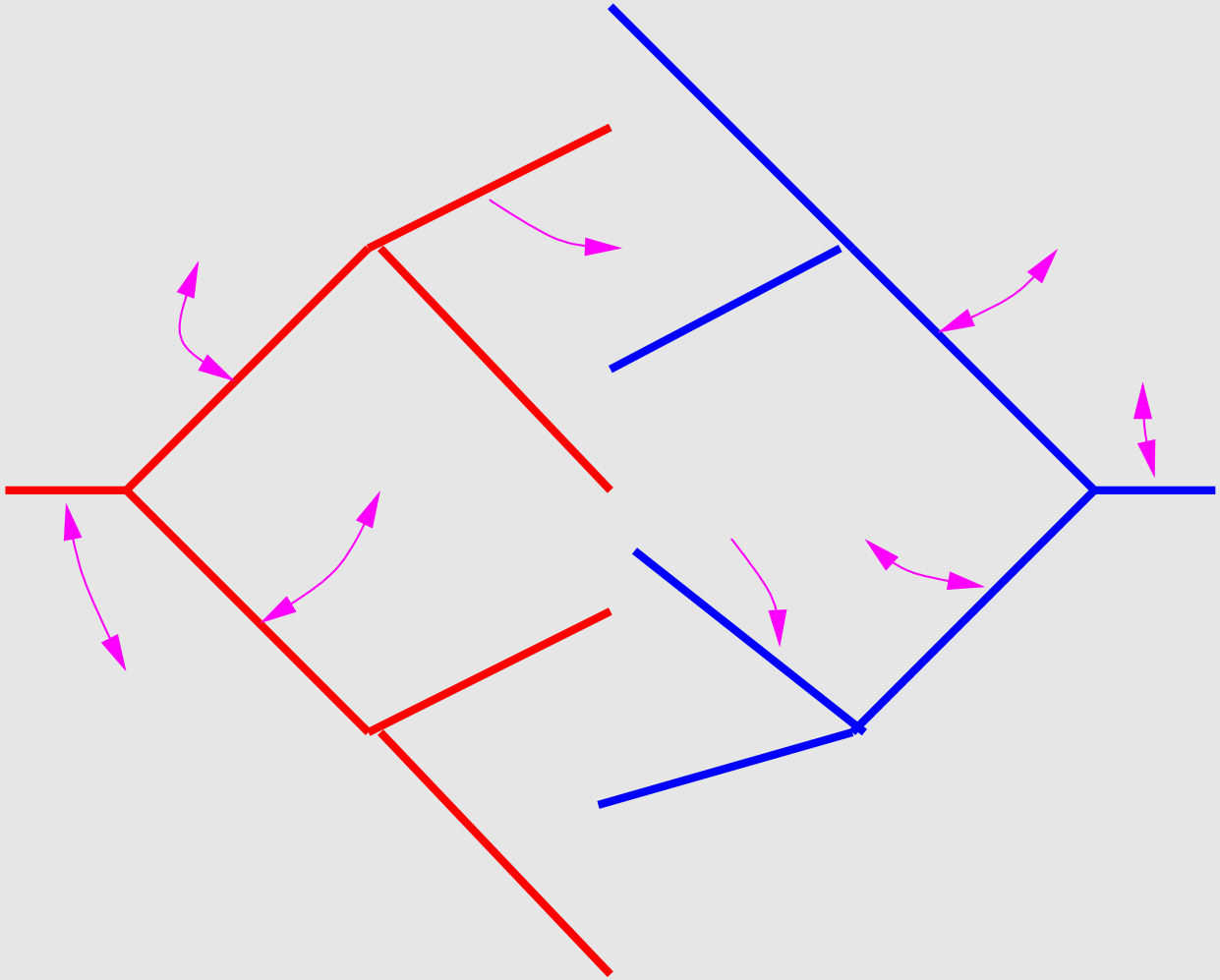
Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

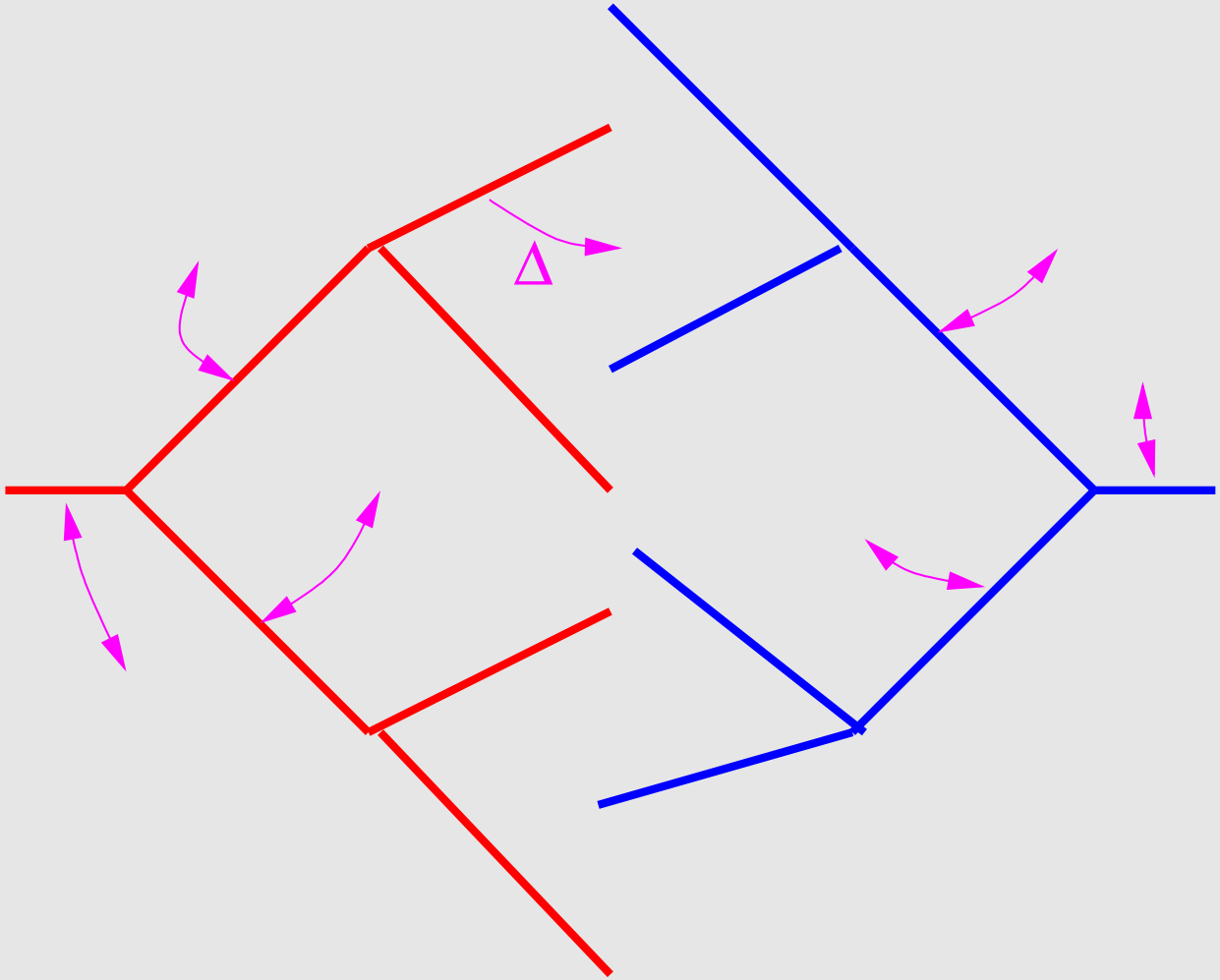
Once more: mass flow



Exchange of Energy, seen by Vessels



Exchange of Energy, seen by Tissue



Heat Transmittivity

non-terminal segments heat up surrounding tissue "diffusively"

$$\kappa_{\text{trans}} = \frac{\text{power (heating tissue)}}{\Delta\text{temperature} \cdot \text{volume of tissue}} \tag{3}$$

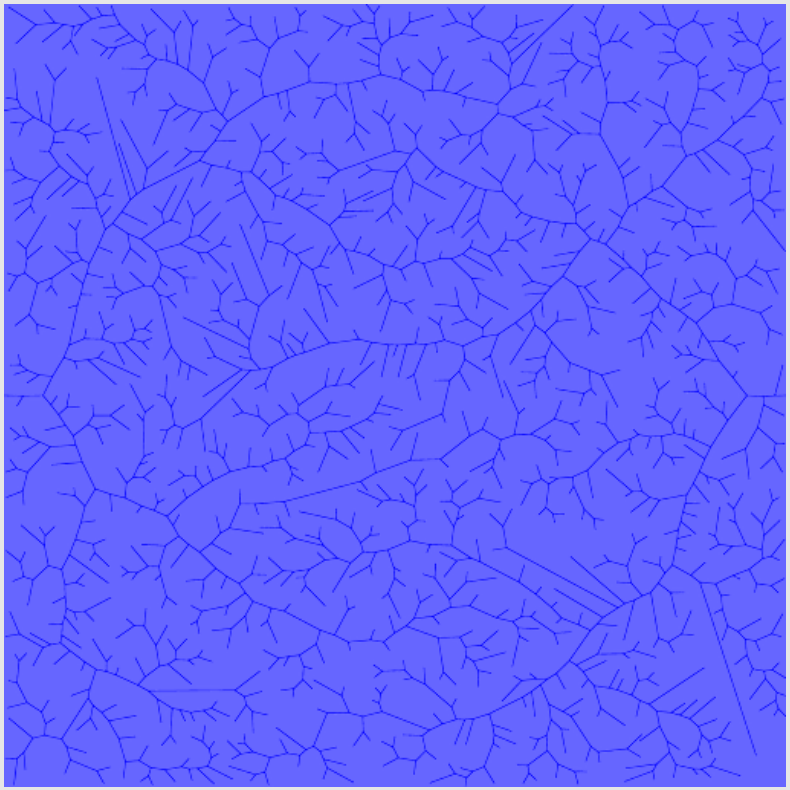
Mutual Source Terms

- terminal arterial segments:
  - vessel sink as before
  - tissue source proportional to Δ temp
- terminal venous segments:
  - vessel source as before
  - no tissue sink
- non-terminal segments:
  - vessel source/sink proportional to Δ temp
  - corresponding tissue sink/source locally conserving energy
- RF probe only heats tissue

Pulses of Inflowing Warm Blood

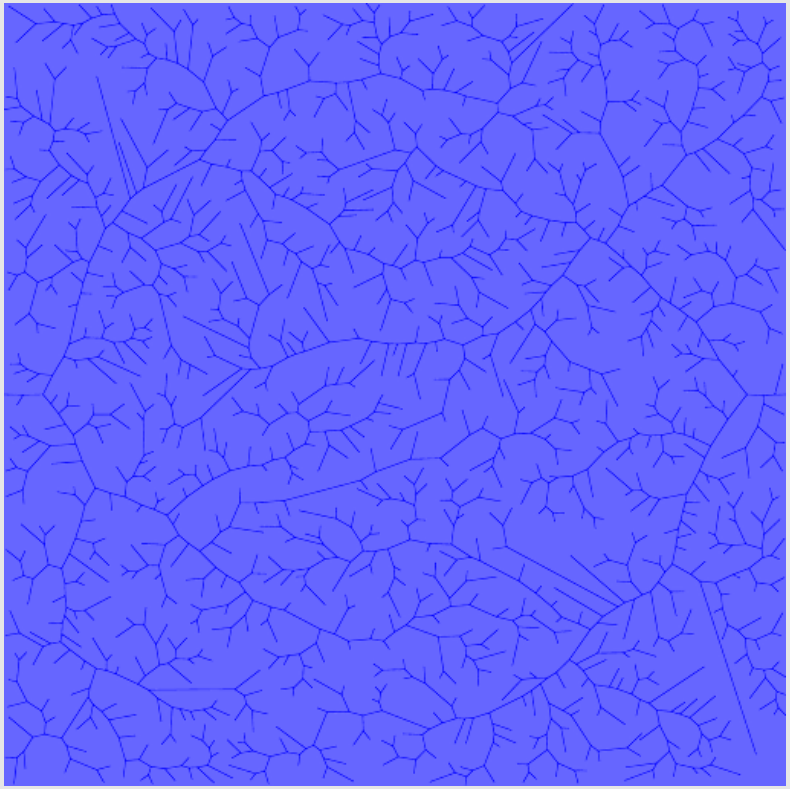
Artificial problem:

- inflowing pulses of warm blood
- pulsed flow velocities.
- different color scales for tissue / vessel temperature



Pulses of Inflowing Warm Blood

Now place RF probe near arterial vessel.
Same color scale for tissue / vessel up to opacity.



Outlook

- 3D
- more complicated geometric structure
- diffusion within vessels
- advection within tissue
- more detailed model of physical effects (vessel walls, non-laminar flow within vessels, thermal influence of RF probe)
- nonlinear dependence of material parameters on temperature, irreversibility
- finer discretization: higher resolution
- comparison to experimental results

Summary

2D Vessel Trees
- Geometric Structure
- Flow Velocities

Advection Problem
- Single Segments
- Bifurcations
- Basis and Test Functions at Bifurcations
- Terminal Segments

Coupling to Diffusion Problem
- Heat Conduction in Tissue
- Exchange of Energy

References

- M. A. Celia, T. F. Russell, I. Herrera, R. .E Ewing: An Eulerian-Lagrangian localized adjoint method for the advection-diffusion equation, Advances in Water Resources 1990, 13(4), p 187–206
- W. Schreiner, M. Neumann et al.: The Branching Angles in Computer-Generated Models of Arterial Trees, J. Gen. Physiol., 105:979–989, 1994
- T. Preusser, H.-O. Peitgen et al.: Simulation of Radio-Frequency Ablation using Composite Finite Element Methods. Proceedings of the Scientific Workshop Medical Robotics, Navigation and Visualization, Rhein-Ahr-Campus Remagen 2004.
- L. O. Schwen: Numerical Simulation of Transport and Diffusion in Drainage Media, Diplomarbeit, Universität Duisburg-Essen, November 2005