Fraunhofer MEVIS

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SOME USE CASES FOR COMPOSITE FINITE ELEMENTS IN IMAGE BASED COMPUTING

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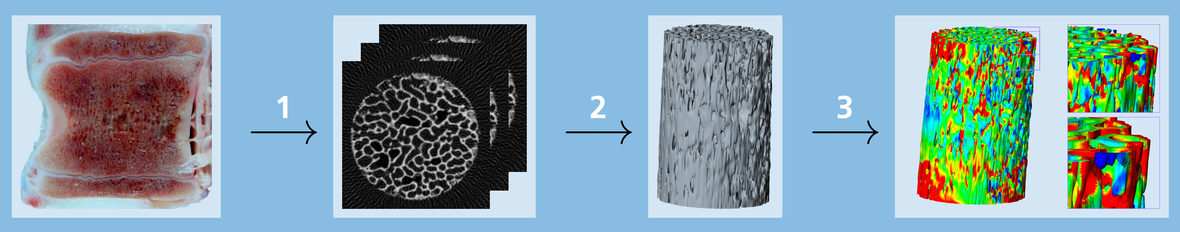
IMAGE-BASED COMPUTING

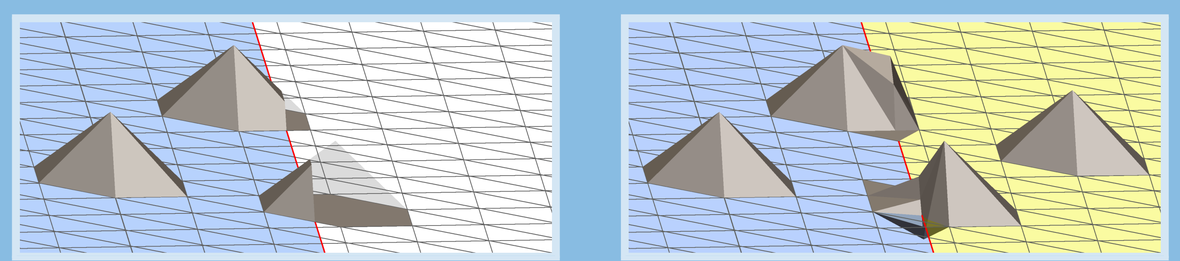


1. Image acquisition (e.g., µCT scan)
2. Image processing (segmentation)
3. Simulation based on voxel grid and visualization

COMPOSITE FINITE ELEMENTS (CFE)

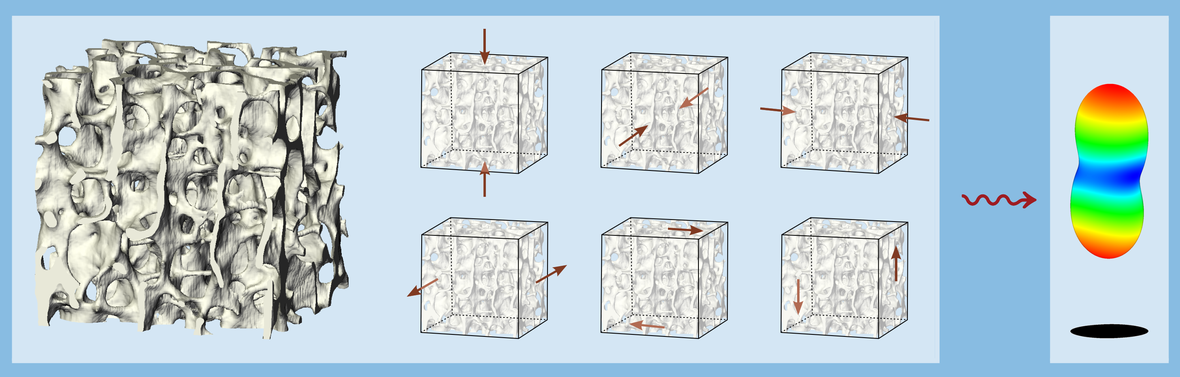
Discretization by CFE basis functions

- Degrees of freedom on (regular) voxel grid
- Information is coded in basis functions instead of modifying the computational grid
- Adapted to sub-voxel accurate approximation of geometrically complicated
  - domain boundary *(left)* [2]
  - interface between different material coefficients *(right)* [4]



- Natural coarse scales permit
  - multigrid solvers
  - numerical homogenization
- Open source implementation available [1]

NUMERICAL HOMOGENIZATION



- Apply unit strains to representative elementary volume
- Evaluate stress, excluding suitable boundary layer [5]
- Resulting effective elasticity tensor (visualization)

VAPORIZATION IN RADIO-FREQUENCY ABLATION

Minimally invasive technique for lesion treatment [3]

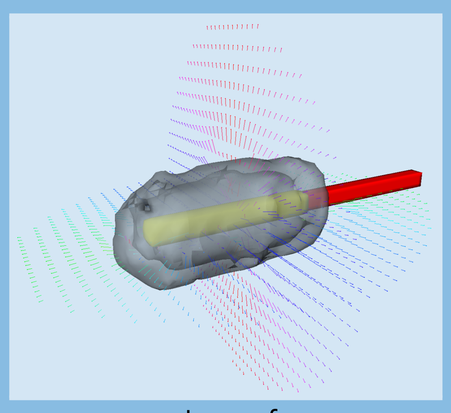
MODEL

- Heating ⇒ vaporization ⇒ liquid and gas phases
  moving interface described by Stefan problem $\frac{[-\lambda\nabla T\cdot N]}{L} = \rho_l(v_l - D)$
- Electrostatic equation $-\operatorname{div}(\sigma\nabla\Phi) = 0$ provides
  heat source $Q_{\text{rf}} = \sigma\|\nabla\Phi\|^2$
- Heat transfer via

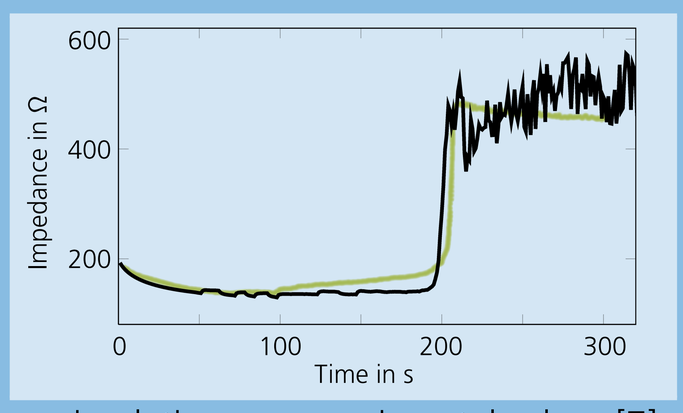
$$\begin{aligned} \rho_g c_g \partial_t T + \rho_g c_g V\cdot\nabla T &= -\operatorname{div}(\lambda_g\nabla T) + Q_{\text{rf}} && \text{in } \Omega_g\times\mathbb{R}^+ \\ \rho_l c_l \partial_t T &= -\operatorname{div}(\lambda_l\nabla T) + Q_{\text{rf}} && \text{in } \Omega_l\times\mathbb{R}^+ \end{aligned}$$

- Electric, thermal conductivities σ, $\lambda_\star$ discontinuous at l/g interface

RESULTS AND VALIDATION



expansion of vapor

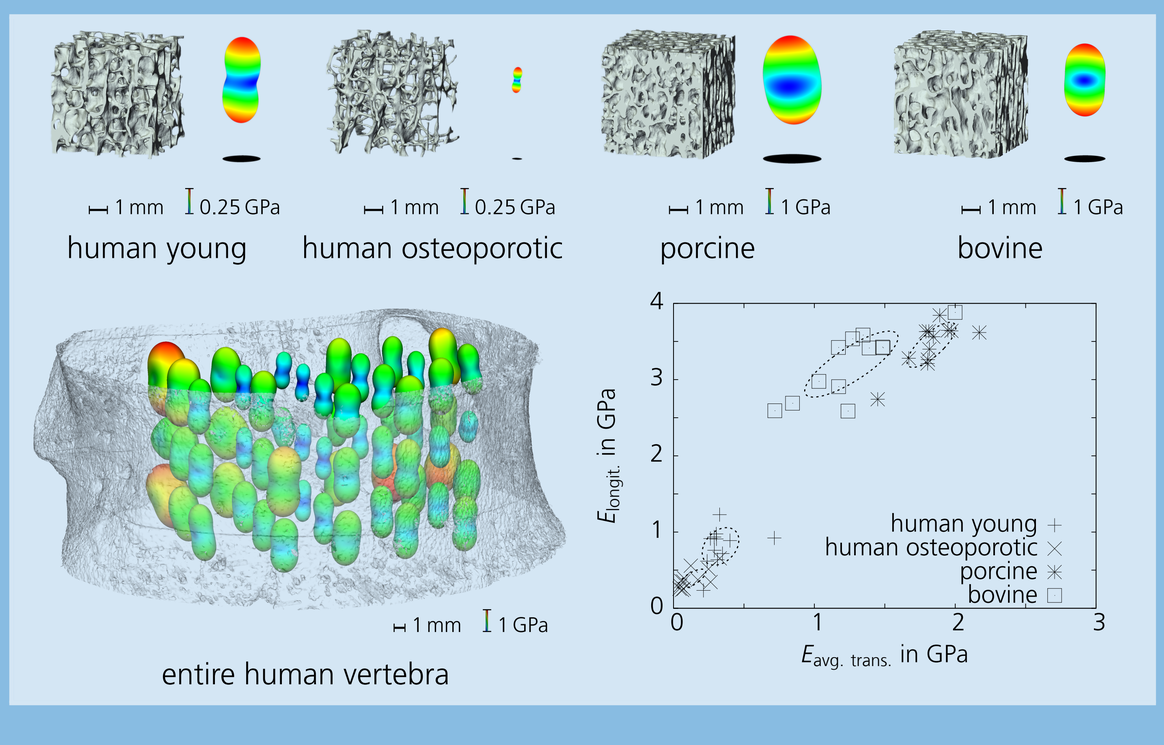


– simulation – experimental values [7]

ELASTIC DEFORMATION OF TRABECULAR BONE

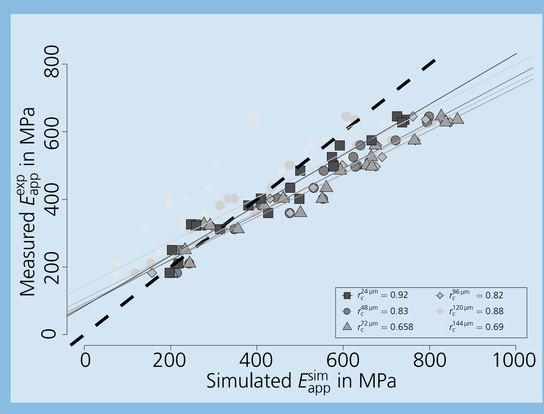
- Assumed isotropic linear elasticity at microscale
  ($E = 13\,\text{GPa}$, $\nu = 0.32$)
- Geometrically complicated bone/void interface
- Determined macroscopic elasticity tensor for cubic specimens [5, 6]

NUMERICAL HOMOGENIZATION RESULTS



VALIDATION

- Uniaxial compressive stiffness: experiment vs. simulation [6]
- Different µCT image = mesh resolutions



REFERENCES

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