

The Expected Running Time of Hierarchical Collision Detection



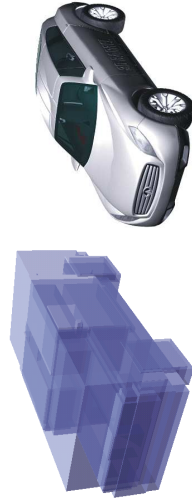
SIGGRAPH2005

Problem

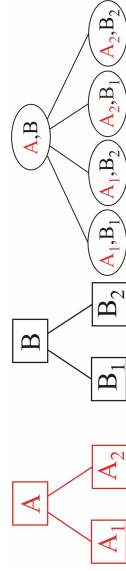
Analysis of the expected running time of hierarchical collision detection that utilizes bounding volume hierarchies.

Until now: $T(n) = N_v C_v + N_p C_p$.

N_v (= num. overlap tests) defines the asymptotic running time.

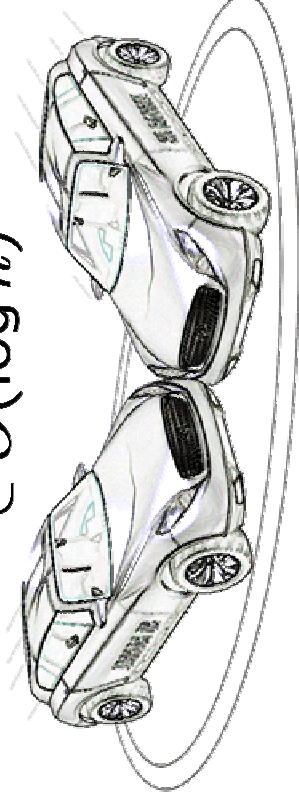


Bounding volume (BV) hierarchy of a model.



Hierarchies for 2 objects. BV test tree.

$\in O(\log n)$



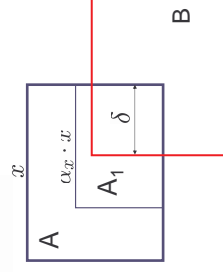
Solution

Compute probability of BV overlap based on two parameters:

- δ : the overlap of the root bounding volumes
- $\alpha_x, \alpha_y, \alpha_z$: the bounding volume diminishing factor.

$$\tilde{N}_v(n) = \sum_{l=1}^{\lg n} \tilde{N}_v^{(l)}(\delta, \alpha_x) \cdot \alpha_y^l \cdot \alpha_z^l$$

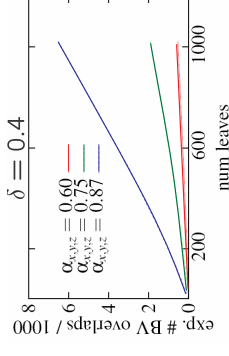
Expected number of overlap tests in BV test tree.



Results

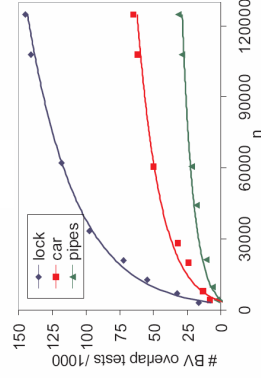
For realistic cases: average running time is in $O(n)$ or even in $O(\log n)$, $n = \#$ leaves.

$\alpha_x, \alpha_y, \alpha_z$	$T(n)$
< 0.5	$O(1)$
0.5	$O(\lg n)$
0.59	$O(\sqrt{n})$
0.71	$O(n)$
0.87	$O(n^{1.58})$
1	$O(n^2)$



Left: Running time $T(n)$ for different BV diminishing factors $\alpha_x, \alpha_y, \alpha_z$ (δ is arbitrary).

Right: Running time for root overlap $\delta = 0.4$



Logarithmic running time for different objects.