Supplemental Material for the Siggraph Sketch
“Nice and Fast Implicit Surfaces over Noisy Point Clouds”

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Figure 1: Visualization of the implicit function \( f(x) \) over a 2D point cloud. The kernel for the moving least squares is based on Euclidean distance or, resp., one of the proximity graphs shown in Figure 2. Points \( x \in \mathbb{R}^2 \) with \( f(x) \approx 0 \), i.e., points on or close to the surface, are shown magenta. Red denotes \( f(x) \gg 0 \) and blue denotes \( f(x) \ll 0 \). (a) point cloud; (b) reconstructed surface using a Euclidean kernel and the covariance matrix defined as \( B_{ij} = \sum_{k=1}^{N} \theta(\|x - p_k\|)(p_{ki} - x_i)(p_{kj} - x_j) \); (c) utilizing a covariance matrix centered at \( a(x) \) instead of \( x \) produces a better surface, but it still has several artifacts; (d) surface and function \( f(x) \) based on our more geodesic kernel using the sphere-of-influence graph.

Figure 2: Different proximity graphs. (a) Delaunay graph \( \text{DG}(\mathcal{P}) \), (b) \( \text{DG}(\mathcal{P}) \) where edges are pruned according to a global sample density, (c) pruning by first quartile, (d) pruning by second quartile, (e) sphere-of-influence graph \( \text{SIG}(\mathcal{P}) \).

Figure 3: If the proximity graph is too thin or too dense, artifacts can occur. On the one hand, we prune the standard Delaunay graph. On the other hand, we augment the standard \( \text{SIG}(\mathcal{P}) \) by edges, in order to prevent too many, unreasonably unconnected components. Therefore, we increase the influence of the nodes in the graph based on the radius \( r \) of the \( k \)-th nearest neighbor, which reflects local point density. Top row: (a) \( \text{DG}(\mathcal{P}) \) where edges are pruned by second quartile, (b) \( 1 - \text{SIG}(\mathcal{P}) \), (c) \( 2 - \text{SIG}(\mathcal{P}) \), (d) \( 3 - \text{SIG}(\mathcal{P}) \). In our experience, \( k = 3 \) or \( k = 4 \) has always worked quite well. Bottom row: the surfaces resulting from these proximity graphs.
Figure 4: Root mean square error (RMSE) for a noisy point cloud (left: original surface). (a) $DG(\mathcal{P})$ with edges larger than second quartile are pruned, (b) $2 - SIG(\mathcal{P})$, (c) Euclidean distance kernel, (d) same with reduced bandwidth $h$, (e) Euclidean distance kernel with optimal bandwidth $h$ that yielded the minimum RMSE; notice the inferior surface quality.

Figure 5: Left: RMSE depending on the kernel bandwidth ($h$) of the points. The $4 - SIG(\mathcal{P})$ allows for the best results (lowest RMSE). Right: (a) original surface, (b) corresponding noisy point cloud.

Figure 6: Average evaluation time of $f(x)$ depending on the kernel bandwidth $h$ (size of point cloud: 1500 points). The timings for 3-SIG and DG are nearly identical (therefore, we omit one curve). Please note that our implementation is not yet fully optimized.
Figure 7: More examples. Left: Euclidean kernel; right: egodesic kernel.

Figure 8: More examples. Left: Euclidean kernel; right: egodesic kernel.